Exercise 1.2 Class 10 Maths CH- 1 Real Numbers Solutions

(From New NCERT book) Session 2023 - 24

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1. Prove that √5 is an irrational number.

Solution: Let us assume that √2 is rational. Then, it will be of the form where a, and b are integers and b ≠ 0.
Again, let a and b have no common factor other than 1.
√5 = a/b, where a and b are coprime integers.

On squaring both sides, we get
$5 = a^2/b^2$
$\Rightarrow 5b^2 = a^2$ ……(1)
$\Rightarrow$5 divides a^2 $\Rightarrow$ 5 divides a **[from Theorem 1]**
Then, a can be written as 5m, where m is an integer.

On putting a = 5m in Eq. (1), we get
$5b^2 = (5m)^2$
$\Rightarrow 5b^2 = 25m^2$
$\Rightarrow b^2 = 5m^2$
So, 5 divides b^2 $\Rightarrow$ 5 divides a **[from Theorem 1]**
Thus, 5 is a common factor of a and b.

But this contradicts the fact that a and b have no common factor other than 1. The contradiction arises by assuming that $\sqrt{5}$ is rational.

Hence, $\sqrt{5}$ is irrational.

Hence proved.

2.Prove that 3 + 2$\sqrt{5}$ is irrational.

Solution:

Let us assume that $3 + 2\sqrt{5}$ is rational. Then it will be of the form a/b where a, b are coprime and ($b \neq 0$)

Now, $3 + 2\sqrt{5} = a/b$

Rearranging this equation, we get

$\sqrt{5} = ½(a/b - 3)$

Since a and b are integers, we get $½(a/b - 3)$ is rational, so their difference will be rational i.e $\sqrt{5}$ is rational.

We know that $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $3 + 2\sqrt{5}$ is
rational.

Hence,$3 + 2\sqrt{5}$ is irrational.

3. Prove that the following are irrational.

(i) $1/\sqrt{2}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

(i) Solution:

Let us assume that $1/\sqrt{2}$ is rational. Then it will be of the form a/b where a, b are coprime and ($b \neq 0$)

Now, $1/\sqrt{2} = a/b$

On rationalising,

$(1 \times \sqrt{2}) / (\sqrt{2} \times \sqrt{2}) = a/b$

$\sqrt{2}/2 = a/b$

Rearranging this equation, we get

$\sqrt{2} = 2a/b$

Since a and b are integers, we get $2a/b$ is rational, This implies that rational i.e $\sqrt{2}$ is rational..

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $1/\sqrt{2}$ is rational.

Hence, $1/\sqrt{2}$ is irrational.

(ii) Solution:

Let us assume that $7\sqrt{5}$ is rational. Then it will be of the form a/b where a, b are coprime and $(b \neq 0)$

Now, $7\sqrt{5} = a/b$

Rearranging this equation, we get

$\sqrt{5} = a/7b$

Since a and b are integers, we get $a/7b$ is rational, This implies that rational i.e $\sqrt{5}$ is rational..

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $7\sqrt{5}$ is rational.

Hence, $7\sqrt{5}$ is irrational.

(iii) Solution:

Let us assume that $6 + \sqrt{2}$ is rational. Then it will be of the form a/b where a, b are coprime and $(b \neq 0)$

Now, 6 + √2 = a/b

Rearranging this equation, we get

√2 = (a/b - 6)

Since a and b are integers, we get (a/b - 6) is rational, so their difference will be rational i.e √2 is rational.

We know that √2 is rational.

But this contradicts the fact that √2 is irrational.

This contradiction has arisen because of our incorrect assumption that 6 + √2 is rational.

Hence, 6 + √2 is irrational.