📘 CLASS 9 MATHEMATICS – CHAPTER 3 NOTES

🌍 The World of Numbers

🆕 New NCERT Ganita Manjari | Grade 9 | Part I | 2026–27

These notes are prepared **directly from the new Ganita Manjari Chapter 3 textbook** covering the chapter from the need to count and the history of numbers through zero, integers, rational and irrational numbers, real numbers, decimal expansions, cyclic numbers and the introduction to imaginary numbers.

🌟 CHAPTER 3: THE WORLD OF NUMBERS

🎯 Learning Objectives

After studying this chapter, you should be able to:

✅ Understand the evolution of numbers
✅ Explain the need for counting numbers
✅ Understand Natural Numbers and Integers
✅ Understand the importance of **zero (Śhūnya)**
✅ Apply rules of operations with integers
✅ Define Rational Numbers
✅ Perform operations on Rational Numbers
✅ Represent rational numbers on the number line
✅ Understand absolute value
✅ Find rational numbers between two rational numbers
✅ Understand Irrational Numbers
✅ Prove that $\sqrt{2}$ is irrational
✅ Construct $\sqrt{2}$ on the number line
✅ Understand the irrationality of π
✅ Understand Real Numbers

✅ Identify terminating and repeating decimals
✅ Convert repeating decimals into fractions
✅ Understand cyclic numbers
✅ Distinguish rational and irrational decimal expansions
✅ Get an introductory idea of imaginary numbers

📚 CHAPTER AT A GLANCE

3.1 🦴 The Dawn of Mathematics: The Human Need to Count

3.2 0️⃣ The Revolution of Śhūnya: When Nothing Became Something

3.3 ➕➖ Integers: Expanding the Horizon

3.4 🍕 Fractions and Rational Numbers

3.5 √ Irrational Numbers

3.6 🌈 Real Numbers: Decimals and Cyclic Patterns

3.7 🚀 Conclusion: The Never-Ending Journey

The chapter contains **Exercise Sets 3.1 to 3.5** followed by end-of-chapter exercises.

🦴 1. THE DAWN OF MATHEMATICS – THE HUMAN NEED TO COUNT

Mathematics began with a very simple human need:

👉 COUNTING

Early humans needed to know:

🐄 How many animals they had
🌾 How much food they had

🏺 How many objects they owned
🌙 How much time had passed
📦 How many goods were being exchanged

The textbook explains this using the idea of **one-to-one correspondence**.

🔢 2. ONE-TO-ONE CORRESPONDENCE

Imagine an ancient herder has a herd of cattle.

For every cow that leaves:

🐄 → 🪨 one pebble

When the cows return:

🐄 → 🪨 remove one pebble

If all pebbles are removed:

✅ All cattle have returned.

If some pebbles remain:

⚠️ Some cattle are missing.

This simple matching process helped humans develop the idea of **numbers**.

⭐ 3. NATURAL NUMBERS

The basic counting numbers are called:

Natural Numbers

They are represented by:

$$N = \{1, 2, 3, 4, 5, ...\}$$

📌 Natural numbers continue indefinitely.

Examples:

```
1, 2, 15, 100, 1000
```

are natural numbers.

🦴 4. HISTORY WRITTEN IN BONE

The chapter connects the development of numbers with ancient tally marks.

🦴 Lebombo Bone

The Lebombo Bone has approximately **29 deliberately carved notches** and is presented in the textbook as evidence of very early counting or tracking of time.

🦴 Ishango Bone

The Ishango Bone contains groups of tally marks.

One column contains:

```
11, 13, 17, 19
```

These are all:

⭐ Prime Numbers between 10 and 20

The textbook also notes another column suggesting doubling/multiplication by 2.

🇮🇳 5. INDIAN CONTRIBUTIONS TO NUMBER SYSTEM

The new textbook gives significant attention to India's mathematical heritage.

Ancient Indian civilisations used:

⚖️ Standardised weights
📏 Standardised measures
💰 Accounting systems
🔢 Large numbers
🔟 Powers of 10
📍 Place-value ideas

Trade centres such as **Lothal and Harappa** required reliable systems of measurement and accounting.

🔟 6. LARGE NUMBERS AND POWERS OF 10

The chapter discusses the ancient Indian interest in very large numbers.

Numbers were expressed using powers of 10.

Examples:

10

$100 = 10^2$

$1000 = 10^3$

$10,000 = 10^4$

and so on.

This development helped prepare the way for the **decimal place-value system**.

0️⃣ 7. THE REVOLUTION OF ŚHŪNYA

One of the greatest developments in mathematics was the concept of:

⭐ ZERO

In Indian mathematics, zero is called:

Śhūnya

The textbook connects the mathematical concept of zero with the philosophical idea of **Śhūnyatā**, meaning emptiness or nothingness.

🧠 8. WHY WAS ZERO IMPORTANT?

Suppose you have:

`5 apples`

and give away all 5.

How many are left?

`5 - 5 = 0`

Zero allows us to represent the absence of a quantity.

It also became essential for the **place-value system**.

For example:

`105`

The zero helps distinguish `105` from `15`.

🇮🇳 9. BRAHMAGUPTA AND ZERO

The textbook highlights the work of:

⭐ Brahmagupta

His work **Brāhmasphuṭasiddhānta (628 CE)** gave explicit rules for arithmetic involving zero.

📜 10. BRAHMAGUPTA'S RULES FOR ZERO

Rule 1️⃣

```
a + 0 = a
```

Adding zero does not change a number.

Example:

```
8 + 0 = 8
```

Rule 2️⃣

```
a - 0 = a
```

Subtracting zero does not change a number.

Example:

```
12 - 0 = 12
```

Rule 3️⃣

```
a × 0 = 0
```

Any number multiplied by zero is zero.

Example:

```
25 × 0 = 0
```

➕➖ 11. INTEGERS – EXPANDING THE HORIZON

Natural numbers were not enough.

What happens when:

`3 - 5`?

The answer is:

-2

This led to the development of **negative numbers**.

The textbook explains positive numbers using the idea of **fortunes (Dhana)** and negative numbers using **debts (Ṛiṇa)**.

🔢 12. INTEGERS

The set of integers is represented by:

Z

Integers include:

`..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...`

So:

Natural Numbers ⊂ Integers

💰 13. POSITIVE AND NEGATIVE NUMBERS

Positive numbers can represent:

💰 Profit
📈 Increase
🌡️ Temperature above zero
🏦 Money you have

Negative numbers can represent:

💸 Debt
📉 Loss
🌡️ Temperature below zero
⬇️ Decrease

➕ 14. ADDITION OF INTEGERS

Same Signs

Positive + Positive

```
(+5) + (+4) = +9
```

Add the numbers and keep the positive sign.

Negative + Negative

```
(-5) + (-4) = -9
```

Add the magnitudes and keep the negative sign.

➖ 15. ADDING INTEGERS WITH DIFFERENT SIGNS

When signs are different:

Step 1

Subtract the smaller absolute value from the larger absolute value.

Step 2

Keep the sign of the number with the larger absolute value.

Example:

```
7 + (-3)

= 7 - 3

= 4
```

Example:

```
(-8) + 3

= -(8 - 3)
```

= −5

✖️ 16. MULTIPLICATION OF INTEGERS

Remember the sign rules:

First	Second	Product
+	+	+
+	-	-
-	+	-
-	-	+

🧠 Easy Trick

Same signs → Positive ✅

Different signs → Negative ❌

Examples:

`(-3) × 4 = -12`

`3 × (-4) = -12`

`(-3) × (-4) = +12`

The textbook explains the negative × negative rule using the idea of removing debts.

➗ 17. DIVISION OF INTEGERS

The same sign rule applies:

Same signs → Positive

`(-20) ÷ (-5) = 4`

Different signs → Negative

`(-20) ÷ 5 = -4`

⭐ 18. SUBTRACTING A NEGATIVE NUMBER

One very important rule is:

`a - (-b) = a + b`

Example:

`10 - (-5)`

`= 10 + 5`

`= 15`

🧠 Why?

Subtracting a debt is equivalent to removing what you owe.

The textbook explicitly asks students to explain this using a debt situation.

🍕 19. FRACTIONS AND RATIONAL NUMBERS

Counting is not enough when we need to describe **parts of a whole**.

Examples:

🍕 Half a pizza → `1/2`

🥛 Half a cup → `1/2`

📏 Three-fourths of a metre → `3/4`

These are fractions.

⭐ 20. RATIONAL NUMBERS

A rational number is any number that can be written in the form:

`p/q`

where:

`p` and `q` are integers

and

`q ≠ 0`

The set of rational numbers is represented by:

`Q`

📌 21. EXAMPLES OF RATIONAL NUMBERS

`1/2`

`-3/4`

`5/7`

`8`

`-10`

`0`

Why are integers rational?

Because:

`8 = 8/1`

`-10 = -10/1`

`0 = 0/1`

Therefore:

Integers ⊂ Rational Numbers

⚠️ 22. WHY CAN'T q = 0?

A rational number is:

```
p/q
```

where:

```
q ≠ 0
```

because division by zero is not defined.

For example:

```
5/0
```

has no defined value.

Therefore:

Denominator can NEVER be zero.

🔄 23. EQUIVALENT RATIONAL NUMBERS

A rational number can have many equivalent forms.

Example:

```
1/2 = 2/4 = 3/6 = 5/10
```

Similarly:

```
-1/3 = -2/6 = -3/9
```

All represent the same number.

The textbook explains that fractions can be simplified by dividing numerator and denominator by a common factor.

✂️ 24. STANDARD / LOWEST FORM

A rational number is generally represented in its simplest form, where numerator and denominator have no common factor other than 1.

Example:

```
12/30
```

Divide by 6:

```
12 ÷ 6 = 2
```

```
30 ÷ 6 = 5
```

Therefore:

`12/30 = 2/5`

➕ 25. ADDITION OF RATIONAL NUMBERS

For:

```
a/b + c/d
```

take a common denominator.

Example:

```
1/3 + 1/6
```

LCM of 3 and 6 = 6.

```
1/3 = 2/6
```

Therefore:

```
2/6 + 1/6
```

= 3/6 = 1/2

➖ 26. SUBTRACTION OF RATIONAL NUMBERS

Example:

5/6 - 1/4

LCM of 6 and 4 = 12.

5/6 = 10/12

1/4 = 3/12

Therefore:

10/12 - 3/12

= 7/12

✖️ 27. MULTIPLICATION OF RATIONAL NUMBERS

Formula:

a/b × c/d = ac/bd

provided denominators are non-zero.

Example:

2/3 × 5/7

= (2 × 5)/(3 × 7)

= 10/21

➗ 28. DIVISION OF RATIONAL NUMBERS

To divide by a fraction, multiply by its reciprocal.

Formula:

`a/b ÷ c/d = a/b × d/c`

Therefore:

`a/b ÷ c/d = ad/bc`

provided the divisor is non-zero.

Example:

`2/3 ÷ 5/7`

`= 2/3 × 7/5`

= 14/15

The textbook gives these operations and the conditions under which they are valid.

🔗 29. PROPERTIES OF RATIONAL NUMBERS

Commutative Property of Addition

`a + b = b + a`

Example:

`2/3 + 1/3 = 1/3 + 2/3`

Commutative Property of Multiplication

$a \times b = b \times a$

Distributive Property

$$a(b + c) = ab + ac$$

Example:

$2(3 + 4)$

$= 2 \times 3 + 2 \times 4$

$= 6 + 8$

$= 14$

🔒 30. CLOSURE PROPERTY

Rational numbers are **closed** under:

✅ Addition
✅ Subtraction
✅ Multiplication
✅ Division, provided we do not divide by zero.

That means performing these operations on rational numbers gives another rational number, subject to the division restriction.

📍 31. RATIONAL NUMBERS ON THE NUMBER LINE

Rational numbers can be represented on the number line.

For example:

`1/2`

lies halfway between:

`0` and `1`

`-3/4`

lies between:

`-1` and `0`

The textbook explains that to represent `p/q`, divide a unit interval into `q` equal parts and move `p` parts in the appropriate direction.

📏 32. REPRESENTING `3/4`

To represent:

`3/4`

Step1️⃣

Take the interval from `0` to `1`.

Step2️⃣

Divide it into 4 equal parts.

Step 3️⃣

Move 3 parts to the right.

The point reached represents:

3/4

📍 33. REPRESENTING A NUMBER GREATER THAN 1

Consider:

9/4

Convert:

$9/4 = 2\ 1/4$

Therefore it lies between:

2 and 3

Divide the interval from 2 to 3 into four equal parts and move one part beyond 2.

📏 34. ABSOLUTE VALUE

The absolute value of a number is its **distance from zero** on the number line.

It is written using vertical bars:

$|x|$

Examples:

$|5| = 5$

`|-5| = 5`

`|0| = 0`

For rational numbers:

`|5/3| = 5/3`

`|-5/3| = 5/3`

Therefore:

`|x| ≥ 0`

Absolute value can never be negative.

📏 35. DISTANCE BETWEEN TWO RATIONAL NUMBERS

For two rational numbers `a` and `b`:

`Distance = |a - b|`

Example:

Distance between `-4` and `3`:

`= |-4 - 3|`

`= |-7|`

`= 7 units`

🌟 36. DENSITY OF RATIONAL NUMBERS

This is one of the most important concepts in the chapter.

Rational numbers are dense.

This means:

> Between any two rational numbers, there is another rational number.

In fact:

There are infinitely many rational numbers between any two rational numbers.

🧮 37. FINDING A RATIONAL NUMBER BETWEEN TWO RATIONAL NUMBERS

A very easy method is to take their average.

If `a` and `b` are two rational numbers:

Rational number between them = `(a + b)/2`

Example:

Find a rational number between:

`1` and `3/2`

Take average:

`(1 + 3/2)/2`

`= (5/2)/2`

`= 5/4`

So:

`1 < 5/4 < 3/2`

The textbook uses this method to demonstrate the density of rational numbers.

♾️ 38. INFINITELY MANY RATIONAL NUMBERS

Suppose we have:

1 and 2

One rational number between them:

3 / 2

Now between:

1 and 3 / 2

we can find:

5 / 4

And between:

3 / 2 and 2

we can find another.

This process can continue forever.

Therefore:

♾️ Infinitely many rational numbers exist between any two rational numbers.

√ 39. IRRATIONAL NUMBERS

Now comes a major question:

❓ Do rational numbers completely fill the number line?

The answer is:

❌ No.

There are numbers that **cannot be expressed as a ratio of two integers**.

These are called:

⭐ IRRATIONAL NUMBERS

Examples:

$\sqrt{2}$

$\sqrt{3}$

$\sqrt{5}$

π

The textbook introduces irrational numbers through the diagonal of a unit square.

📐 40. WHY IS √2 IRRATIONAL?

Consider a square whose side is:

`1 unit`

Using Pythagoras:

$d^2 = 1^2 + 1^2$

$d^2 = 2$

Therefore:

$\mathbf{d = \sqrt{2}}$

This diagonal has length $\sqrt{2}$.

But $\sqrt{2}$ cannot be expressed as:

p/q

where p and q are integers.

Therefore:

$\sqrt{2}$ is irrational.

🧠 41. PROOF BY CONTRADICTION

The textbook uses an important mathematical method called:

Proof by Contradiction

The basic idea is:

1️⃣ Assume the opposite of what you want to prove.

2️⃣ Use logical reasoning.

3️⃣ Arrive at a contradiction.

4️⃣ Therefore, the original assumption must be false.

✏️ 42. PROOF THAT √2 IS IRRATIONAL

Suppose:

$$\sqrt{2} = p/q$$

where p/q is in lowest form.

So p and q have no common factor other than 1.

Step 1: Square both sides

$2 = p^2/q^2$

Step 2: Multiply by q^2

$2q^2 = p^2$

Therefore p^2 is even.

If the square of a number is even, the number itself must be even.

Therefore:

p is even.

Let:

$p = 2k$

Step 3: Substitute

$2q^2 = (2k)^2$

$2q^2 = 4k^2$

Divide by 2:

$q^2 = 2k^2$

Therefore q^2 is even.

So:

q is even.

🚨 Step 4: Contradiction

We found that:

p is even

and

`q` is even.

Therefore both have a common factor:

`2`

But initially we assumed that `p/q` was in lowest form.

This is a contradiction.

Therefore our original assumption was false.

⭐ Hence, √2 is irrational.

This is the complete logical structure used in the textbook.

📐 43. CONSTRUCTING √2 ON THE NUMBER LINE

The textbook also shows a geometric construction.

Step 1️⃣

Take a number line and mark:

```
O = 0
```

```
A = 1
```

So:

```
OA = 1 unit
```

Step 2️⃣

At A, draw a perpendicular.

Mark:

$AB = 1$ unit

Step 3️⃣

Join O to B.

Using Pythagoras:

$$OB^2 = OA^2 + AB^2$$

$$= 1^2 + 1^2$$

$$= 2$$

Therefore:

$$OB = \sqrt{2}$$

Step 4️⃣

Using a compass with:

OB

as radius, transfer this length onto the number line.

The resulting point represents:

$\sqrt{2}$

🇮🇳 44. THE STORY OF π

Another famous irrational number is:

π

It represents the ratio:

`Circumference / Diameter`

of a circle.

🇮🇳 45. ĀRYABHAṬA AND π

The textbook discusses Āryabhaṭa's approximation:

$$\pi \approx 3927/1250 = 3.1416$$

This was an approximation rather than an exact fractional representation.

🌟 46. MĀDHAVA'S INFINITE SERIES

The chapter also introduces the famous infinite series associated with **Mādhava of Sangamagrama**:

$$\pi = 4(1 - 1/3 + 1/5 - 1/7 + \dots)$$

This shows how an irrational number can be represented through an **infinite process** rather than a single fraction.

🌈 47. REAL NUMBERS

When we combine:

Rational Numbers + Irrational Numbers

we get:

⭐ REAL NUMBERS

The set of real numbers is represented by:

R

So:

R = Rational Numbers ∪ Irrational Numbers

Real numbers correspond to points on the complete real number line.

📊 48. NUMBER SYSTEM HIERARCHY

The number system can be visualised as:

Natural Numbers
↓
Integers
↓
Rational Numbers
↓
Real Numbers

But remember:

Real Numbers contain BOTH:

🔵 Rational Numbers
🟠 Irrational Numbers

🧠 49. NUMBER SYSTEM INCLUSION

```
N ⊂ Z ⊂ Q ⊂ R
```

That means:

Every natural number is an integer.

Every integer is a rational number.

Every rational number is a real number.

But:

❌ Every real number is NOT rational.

Because irrational numbers are also real numbers.

🔢 50. DECIMAL EXPANSIONS OF RATIONAL NUMBERS

A rational number can have two types of decimal expansions:

1️⃣ Terminating decimal

The decimal ends.

Example:

```
3/8 = 0.375
```

2️⃣ Non-terminating repeating decimal

The decimal continues forever but a block of digits repeats.

Example:

```
5/11 = 0.454545...
```

or:

```
0.45
```

with the repeating block understood.

The textbook derives this classification from long division and the possible remainders.

⭐ 51. TERMINATING DECIMAL

A decimal is terminating if the division eventually gives remainder zero.

Example:

```
3/8
```

```
= 0.375
```

The decimal stops.

🔁 52. REPEATING DECIMAL

A decimal is repeating if a sequence of digits repeats forever.

Example:

```
1/3 = 0.3333...
```

Example:

```
5/11 = 0.454545...
```

The repeating block is:

```
45
```

🔍 53. HOW CAN WE PREDICT THE DECIMAL TYPE?

Suppose:

p/q

is in lowest form.

Look at the **prime factors of q**.

If q has only factors 2 and/or 5:

✅ Decimal terminates.

If q has any prime factor other than 2 or 5:

🔁 Decimal is non-terminating and repeating.

The textbook gives this criterion explicitly.

🧮 54. EXAMPLES

Example 1

$3/20$

Prime factorisation:

$20 = 2^2 \times 5$

Only 2 and 5 occur.

Therefore:

✅ Terminating

$3/20 = 0.15$

Example 2

`7/25`

$25 = 5^2$

Only 5 occurs.

Therefore:

✅ Terminating

`7/25 = 0.28`

Example 3

`4/15`

$15 = 3 \times 5$

There is a factor `3`.

Therefore:

🔁 Non-terminating repeating

📝 55. QUICK DECIMAL RULE

Denominator in lowest form:

Only 2 and/or 5 → Terminating ✅

Any other prime factor → Repeating 🔁

🔄 56. CONVERTING A TERMINATING DECIMAL TO p/q

Example:

`0.35`

Write over 100:

`0.35 = 35/100`

Simplify:

`= 7/20`

Therefore:

`0.35 = 7/20`

This is the textbook's example.

🔁 57. CONVERTING A PURE REPEATING DECIMAL

Consider:

`0.6 = 0.6666...`

Let:

`x = 0.6666...`

Multiply by 10:

`10x = 6.6666...`

Subtract:

`10x - x = 6.6666... - 0.6666...`

`9x = 6`

Therefore:

`x = 6/9`

`x = 2/3`

So:

`0.6 = 2/3`

🔁 58. TWO-DIGIT REPEATING DECIMAL

Consider:

`0.45 = 0.454545...`

Let:

`x = 0.454545...`

Because two digits repeat, multiply by 100:

`100x = 45.454545...`

Subtract:

`100x - x = 45`

`99x = 45`

Therefore:

`x = 45/99`

Simplify:

`x = 5/11`

🔄 59. GENERAL REPEATING DECIMAL

A general repeating decimal has:

➡️ Some non-repeating digits

followed by:

🔁 A repeating block.

Example:

$$0.16$$

where 6 repeats.

Let:

$$x = 0.16666...$$

First shift the non-repeating digit:

$$10x = 1.6666...$$

Then shift one complete repeating cycle:

$$100x = 16.6666...$$

Subtract:

$$100x - 10x = 15$$

$$90x = 15$$

Therefore:

$$x = 15/90$$

$$x = 1/6$$

🧮 60. GENERAL METHOD

Suppose:

`x = decimal`

with:

`m` = number of non-repeating digits

`n` = number of repeating digits

Then:

1️⃣ Multiply by `10^m`.

2️⃣ Multiply again by `10^n`.

3️⃣ Subtract the equations.

4️⃣ Solve for `x`.

This is the method summarised by the textbook.

🔢 61. CYCLIC NUMBERS

One of the most interesting parts of the chapter is:

✨ Cyclic Numbers

Consider:

`1/7 = 0.142857142857...`

The repeating block is:

`142857`

Now multiply it:

```
142857 × 1 = 142857

142857 × 2 = 285714

142857 × 3 = 428571

142857 × 4 = 571428

142857 × 5 = 714285

142857 × 6 = 857142
```

Notice what happened?

🔄 The same digits appear in different cyclic orders.

This is why `142857` is called a **cyclic number**.

🔴 62. IRRATIONAL DECIMALS

Irrational numbers have decimals that:

❌ Never terminate

and

❌ Never repeat a fixed block.

Examples:

```
√2 = 1.41421356237...

π = 3.14159265358...
```

Therefore:

Irrational → non-terminating + non-repeating decimal

⚠️ 63. VERY IMPORTANT COMPARISON

Type	Decimal Expansion
Rational	Terminating OR repeating
Irrational	Non-terminating AND non-repeating
Real	Rational + irrational

🤯 64. `0.999... = 1`

The chapter introduces the surprising fact:

`0.9999... = 1`

Let's prove it.

Let:

`x = 0.9999...`

Multiply by 10:

`10x = 9.9999...`

Subtract:

`10x - x = 9.9999... - 0.9999...`

`9x = 9`

Therefore:

`x = 1`

So:

`0.999... = 1`

The textbook discusses this as an example of the non-uniqueness of decimal representations.

🔄 65. NON-UNIQUENESS OF DECIMAL REPRESENTATION

Some numbers have two decimal representations.

For example:

`1.000... = 0.999...`

Similarly:

`2.47000... = 2.46999...`

Both represent the same real number.

🚀 66. IMAGINARY NUMBERS – A GLIMPSE

The chapter ends by asking:

❓ What is `√(-1)`?

We know:

`1 × 1 = 1`

and:

`(-1) × (-1) = 1`

So no real number squared gives:

`-1`

Mathematicians therefore introduced a new number:

$$i = \sqrt{(-1)}$$

These are called:

⭐ Imaginary Numbers

The textbook gives this only as an introduction and notes their importance in areas such as electrical engineering and quantum mechanics.

📌 **For Class 9, remember the concept; detailed imaginary-number operations come later.**

📊 67. COMPLETE NUMBER SYSTEM

```
           REAL NUMBERS (R)
           /              \
          /                \
    RATIONAL (Q)        IRRATIONAL
       /                   \
      /                     \
  INTEGERS (Z)             √2, √3, π
     /
    /
NATURAL (N)
1, 2, 3, ...
```

Inclusion:

$$N \subset Z \subset Q \subset R$$

and:

$$R = Q \cup \text{Irrational Numbers}$$

🧠 68. IMPORTANT SYMBOLS

Symbol	Meaning
`N`	Natural Numbers
`Z`	Integers
`Q`	Rational Numbers
`R`	Real Numbers
`p/q`	Rational number form
`q ≠ 0`	Denominator cannot be zero
`	x
`√2`	Irrational number
`π`	Irrational number
`i`	Imaginary unit

📚 69. IMPORTANT PROPERTIES

🔹 Natural Numbers

```
N = {1,2,3,...}
```

🔹 Integers

```
Z = {...,-2,-1,0,1,2,...}
```

🔹 Rational Numbers

```
Q = {p/q : p,q ∈ Z, q ≠ 0}
```

🔹 Real Numbers

`R = Rational ∪ Irrational`

🧮 70. IMPORTANT FORMULAS

Rational number

`p/q, q ≠ 0`

Addition

`a/b + c/d = (ad + bc)/bd`

Subtraction

`a/b - c/d = (ad - bc)/bd`

Multiplication

`a/b × c/d = ac/bd`

Division

`a/b ÷ c/d = ad/bc`

Absolute value

`|x| = distance of x from 0`

Distance between a and b

`|a - b|`

Rational number between a and b

`(a + b)/2`

Rational decimal criterion

Denominator in lowest form contains only:

2 and/or 5 → terminating

Any other prime factor → repeating

📊 71. NUMBER SYSTEM REVISION TABLE

Number Type	Definition	Examples
Natural	Counting numbers	`1,2,3,...`
Integer	Positive, negative numbers and zero	`-3,-2,-1,0,1,2,3`
Rational	Can be written as `p/q`	`1/2,-3/4,5`
Irrational	Cannot be written as `p/q`	`√2, π`
Real	Rational + irrational	All points on real number line

The textbook's chapter summary gives the same overall progression from Natural Numbers through Integers, Rational Numbers, Irrational Numbers and Real Numbers.

⚠️ 72. COMMON MISTAKES TO AVOID

❌ Mistake 1

Thinking zero is a natural number.

For this textbook:

```
N = {1,2,3,...}
```

Zero is treated separately before integers.

❌ Mistake 2

Writing a rational number with denominator zero.

❌ **`5/0`**

✅ **`5/1`**

❌ Mistake 3

Thinking every non-terminating decimal is irrational.

Wrong!

Example:

```
1/3 = 0.3333...
```

is rational.

⭐ Non-terminating + repeating → Rational

⭐ Non-terminating + non-repeating → Irrational

❌ Mistake 4

Thinking `√4` is irrational.

```
√4 = 2
```

and 2 is rational.

❌ Mistake 5

Forgetting to reduce a fraction before checking its decimal type.

Always put:

`p/q`

in **lowest form** first.

❌ Mistake 6

Thinking absolute value can be negative.

`|-8| = 8`

Absolute value is always:

`≥ 0`

❌ Mistake 7

Thinking there is only one rational number between two rational numbers.

There are:

♾️ Infinitely many.

🎯 73. EXAM-IMPORTANT CONCEPTS

⭐ Natural Numbers and their historical origin

⭐ Zero and Brahmagupta's rules

⭐ Integers and sign rules

⭐ Rational number definition

⭐ Why denominator cannot be zero

⭐ Equivalent rational numbers

⭐ Operations on rational numbers

⭐ Closure properties

⭐ Number-line representation

⭐ Absolute value

⭐ Density of rational numbers

⭐ Finding rational numbers between two numbers

⭐ Definition of irrational numbers

⭐ Proof of irrationality of $\sqrt{2}$

⭐ Construction of $\sqrt{2}$

⭐ π and Madhava's infinite series

⭐ Real Numbers

⭐ Terminating decimals

⭐ Repeating decimals

⭐ Decimal-to-fraction conversion

⭐ Cyclic numbers

⭐ `0.999... = 1`

⭐ Rational vs irrational decimals

⭐ Introduction to imaginary numbers

🧠 74. COMPETENCY-BASED THINKING

This chapter is especially rich in reasoning questions.

🔍 Identify

Is `0.3333...` rational or irrational?

🧠 Explain

Why must the denominator of a rational number be non-zero?

📍 Represent

Locate `3/4` on a number line.

🧮 Calculate

Find:

```
2/3 + 5/6
```

🔎 Reason

Find three rational numbers between `1/2` and `3/4`.

🧠 Prove

Show that `√2` is irrational.

📊 Analyse

Without long division, decide whether `18/125` has a terminating decimal.

🔄 Convert

Convert `0.45` into `p/q`.

💡 Explain

Why is `0.999... = 1`?

These kinds of tasks align closely with the textbook's exercises, which include proof, construction, number-line representation, decimal classification and reasoning problems.

📝 75. QUICK REVISION SHEET

🔢 Natural Numbers

`N = {1,2,3,...}`

0️⃣ Zero

`a + 0 = a`

`a - 0 = a`

`a × 0 = 0`

➕➖ Integers

`Z = {...,-2,-1,0,1,2,...}`

Same signs in multiplication → `+`

Different signs → `-`

🍕 Rational Numbers

`p/q`, where:

`p,q ∈ Z`

and:

`q ≠ 0`

📍 Absolute Value

`|x| ≥ 0`

📏 Distance

`|a-b|`

♾️ Density

Between any two rational numbers:

infinitely many rational numbers exist.

√ Irrational Numbers

Cannot be expressed as:

`p/q`

Examples:

`√2, √3, π`

🌈 Real Numbers

```
R = Rational + Irrational
```

🔢 Decimal Rule

Only 2 and/or 5 in denominator → terminating

Any other prime factor → repeating

🔁 Repeating Decimal

```
0.666... = 2/3
```

✨ Cyclic Number

```
142857
```

🤯 Special Result

```
0.999... = 1
```

🚀 Imaginary Number

```
i = √(-1)
```

🌟 CHAPTER TAKEAWAY

Chapter 3 takes us on a journey:

🦴 Counting

⬇️

🔢 Natural Numbers

⬇️

0️⃣ Zero

⬇️

➕➖ Integers

⬇️

🍕 Rational Numbers

⬇️

√ Irrational Numbers

⬇️

🌈 Real Numbers

⬇️

🔁 Decimal & Cyclic Patterns

⬇️

🚀 Imaginary Numbers — a glimpse

The central message of the chapter is that the **number system developed gradually to solve increasingly complex problems**. Natural numbers helped us count, zero represented absence, integers handled debts and negative quantities, rational numbers represented fractions and measurements, while irrational numbers revealed that not every quantity can be expressed as a fraction. Rational and irrational numbers together form the Real Number System.

💡 Remember this hierarchy:

$$N \subset Z \subset Q \subset R$$

and

$$R = \text{Rational} + \text{Irrational}$$ 🌍➗√