

CLASS 9 MATHEMATICS – CHAPTER 4 NOTES

Exploring Algebraic Identities

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These notes follow the **new Ganita Manjari Chapter 4**. The chapter focuses on visualising algebraic identities, factorisation, algebra tiles, new identities and simplification of rational algebraic expressions.

CHAPTER 4: EXPLORING ALGEBRAIC IDENTITIES

Learning Objectives

After studying this chapter, you should be able to:

- ✓ Understand what an algebraic identity is
 - ✓ Distinguish between an equation and an identity
 - ✓ Visualise identities using geometrical models
 - ✓ Expand algebraic expressions using identities
 - ✓ Factorise algebraic expressions using identities
 - ✓ Factorise quadratic expressions
 - ✓ Use algebra tiles to understand factorisation
 - ✓ Factorise without algebra tiles
 - ✓ Apply identities to numerical calculations
 - ✓ Derive new identities
 - ✓ Factorise cubic expressions
 - ✓ Simplify rational algebraic expressions
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CHAPTER AT A GLANCE

4.1 Introduction

4.2 Visualising Identities

4.3 Factorisation of Algebraic Expressions Using Identities

4.4 More Identities

4.5 Factorisation Using Algebra Tiles

4.6 Factorisation Without Using Algebra Tiles

4.7 Finding New Identities

4.8 Simplifying Rational Expressions

The chapter begins on textbook page 68 and ends on page 91.



1. WHAT IS AN ALGEBRAIC IDENTITY?

An **algebraic identity** is an equation that is true for **all values of the variables** involved.

Example:

$$(x + y)^2 = x^2 + 2xy + y^2$$

This is true for every value of x and y .

Therefore, it is an:



Algebraic Identity



2. EQUATION VS IDENTITY

◆ Equation

An equation may be true only for particular values.

Example:

$$x^2 - 1 = 24$$

This is true when:

$$x = 5 \text{ or } x = -5$$

Therefore, it is an equation, not an identity.

◆ Identity

Example:

$$(x + y)^2 = x^2 + 2xy + y^2$$

It is true for **all values** of x and y .

 **Remember:**

Equation → true for some values

Identity → true for all values

3. A PATTERN WITH CONSECUTIVE SQUARES

Consider three consecutive square numbers:

$$1, 4, 9$$

Take the first and last:

$$1 + 9 = 10$$

Subtract twice the middle:

$$10 - 2(4)$$

$$= 10 - 8$$

$$= 2$$

Try:

$$9, 16, 25$$

$$9 + 25 - 2(16)$$

$$= 34 - 32$$

$$= 2$$

The result is always 2 for three consecutive square numbers.

4. WHY DOES THIS PATTERN ALWAYS WORK?

Take three consecutive numbers:

$$n - 1, n, n + 1$$

Their squares are:

$$(n - 1)^2, n^2, (n + 1)^2$$

Now:

$$(n - 1)^2 + (n + 1)^2$$

Expand:

$$= n^2 - 2n + 1 + n^2 + 2n + 1$$

$$= 2n^2 + 2$$

Subtract twice the middle square:

$$2n^2 + 2 - 2n^2$$

$$= 2$$

🎯 Hence the result is always 2.



5. VISUALISING ALGEBRAIC IDENTITIES

The chapter uses:



Squares



Rectangles



Algebra tiles



Cubes

to make algebraic identities easier to understand.

For example, consider a square of side:

$$a + b$$

Its area is:

$$(a + b)^2$$

The square can be divided into:

- one square of area a^2
- one square of area b^2
- two rectangles of area ab each

Therefore:

$$(a + b)^2 = a^2 + ab + ab + b^2$$

Hence:

★ $(a + b)^2 = a^2 + 2ab + b^2$

★ **6. FIRST IMPORTANT IDENTITY**

$$(a + b)^2 = a^2 + 2ab + b^2$$

In words:

Square of the sum = square of first + twice the product + square of second

Example:

$$(x + 3)^2$$

Here:

$$a = x$$

$$b = 3$$

Therefore:

$$(x + 3)^2$$

$$= x^2 + 2(x)(3) + 3^2$$

$$= x^2 + 6x + 9$$

 7. IDENTITY WORKS FOR NEGATIVE NUMBERS TOO

Take:

$$a = -2$$

$$b = -3$$

Then:

$$(a + b)^2 = (-5)^2 = 25$$

Right side:

$$a^2 + 2ab + b^2$$

$$= 4 + 12 + 9$$

$$= 25$$

Therefore the identity still works.

It also works for rational numbers.



8. DERIVING THE IDENTITY ALGEBRAICALLY

Start with:

$$(a + b)^2$$

$$= (a + b)(a + b)$$

Using distributive property:

$$= a(a + b) + b(a + b)$$

$$= a^2 + ab + ba + b^2$$

Since:

$$ab = ba$$

we get:

$$(a + b)^2 = a^2 + 2ab + b^2$$



9. VERY IMPORTANT

Do NOT write:

✗ $(a + b)^2 = a^2 + b^2$

The correct expansion is:

✓ $(a + b)^2 = a^2 + 2ab + b^2$

The middle term $2ab$ is essential.



10. EXPANDING ALGEBRAIC EXPRESSIONS

Example:

$$(5x + 2y)^2$$

Use:

$$(a + b)^2 = a^2 + 2ab + b^2$$

Here:

$$a = 5x$$

$$b = 2y$$

Therefore:

$$(5x + 2y)^2$$

$$= (5x)^2 + 2(5x)(2y) + (2y)^2$$

$$= 25x^2 + 20xy + 4y^2$$

★ Answer:

$$25x^2 + 20xy + 4y^2$$



11. USING IDENTITY FOR QUICK CALCULATIONS

Example:

Find 43^2

Write:

$$43 = 40 + 3$$

Therefore:

$$43^2 = (40 + 3)^2$$

$$= 40^2 + 2(40)(3) + 3^2$$

$$= 1600 + 240 + 9$$

$$= 1849$$

This is much faster than direct multiplication.



12. FACTORISATION USING IDENTITIES

Identities can be used in the reverse direction.

For example:

$$x^2 + 4x + 4$$

Notice:

$$x^2 = x^2$$

$$4x = 2(x)(2)$$

$$4 = 2^2$$

Therefore:

$$x^2 + 4x + 4$$

$$= x^2 + 2(x)(2) + 2^2$$

$$= (x + 2)^2$$

Therefore:

$$\star x^2 + 4x + 4 = (x + 2)^2$$



13. PERFECT SQUARE TRINOMIAL

An expression of the form:

$$a^2 + 2ab + b^2$$

can be written as:

$$(a + b)^2$$

Similarly:

$$a^2 - 2ab + b^2$$

can be written as:

$$(a - b)^2$$

These are called **perfect square forms**.

★ 14. SECOND IMPORTANT IDENTITY

$$(a - b)^2 = a^2 - 2ab + b^2$$

Example:

$$(x - 3)^2$$

$$= x^2 - 2(x)(3) + 3^2$$

$$= x^2 - 6x + 9$$



15. QUICK SQUARE USING $(a - b)^2$

Find:

$$29^2$$

Write:

$$29 = 30 - 1$$

Therefore:

$$29^2 = (30 - 1)^2$$

$$= 30^2 - 2(30)(1) + 1^2$$

$$= 900 - 60 + 1$$

$$= 841$$



16. GEOMETRICAL MEANING OF $(a - b)^2$

Take a square of side a .

Its area:

$$a^2$$

If a portion of length b is removed from each side, the smaller square has side:

$$a - b$$

Therefore its area is:

$$(a - b)^2$$

The removed rectangular portions allow us to derive:

$$(a - b)^2 = a^2 - 2ab + b^2$$



17. FACTORISATION EXAMPLE

Factor:

$$36x^2 + 12x + 1$$

Observe:

$$36x^2 = (6x)^2$$

$$12x = 2(6x)(1)$$

$$1 = 1^2$$

Therefore:

$$36x^2 + 12x + 1$$

$$= (6x)^2 + 2(6x)(1) + 1^2$$

$$= (6x + 1)^2$$



18. FACTORISATION WITH A COMMON FACTOR

Consider:

$$50p^2 + 60pq + 18q^2$$

Take 2 common:

$$= 2(25p^2 + 30pq + 9q^2)$$

Now:

$$25p^2 = (5p)^2$$

$$30pq = 2(5p)(3q)$$

$$9q^2 = (3q)^2$$

Therefore:

$$= 2(5p + 3q)^2$$

★ **Answer:**

$$2(5p + 3q)^2$$



19. THREE CONSECUTIVE SQUARES – ALGEBRAIC PROOF

For:

$$n - 1, n, n + 1$$

we have:

$$\begin{aligned} & (n - 1)^2 + (n + 1)^2 - 2n^2 \\ &= (n^2 - 2n + 1) + (n^2 + 2n + 1) - 2n^2 \\ &= 2 \end{aligned}$$

 **Therefore:**

The sum of the smallest and largest of three consecutive squares minus twice the middle square is always 2.

20. MORE IMPORTANT IDENTITIES

The chapter develops several additional identities.

Identity 1

$$(a + b)^2 = a^2 + 2ab + b^2$$

Identity 2

$$(a - b)^2 = a^2 - 2ab + b^2$$

Identity 3

$$\begin{aligned} & (a + b + c)^2 \\ &= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \end{aligned}$$



21. SQUARE OF THREE TERMS

Formula:

$$\begin{aligned}(a + b + c)^2 \\&= a^2 + b^2 + c^2 \\&+ 2ab + 2bc + 2ca\end{aligned}$$

Example:

$$\begin{aligned}(x + y + z)^2 \\&= x^2 + y^2 + z^2 \\&+ 2xy + 2yz + 2zx\end{aligned}$$



22. EXAMPLE – SQUARE OF 119

Write:

$$119 = 100 + 10 + 9$$

Therefore:

$$\begin{aligned}119^2 \\&= 100^2 + 10^2 + 9^2 \\&+ 2(100)(10) + 2(10)(9) + 2(100)(9) \\&= 10000 + 100 + 81 + 2000 + 180 + 1800 \\&= \mathbf{14161}\end{aligned}$$

★ 23. DIFFERENCE OF TWO SQUARES

Another extremely important identity:

$$(a + b)(a - b) = a^2 - b^2$$

Therefore:

$$a^2 - b^2 = (a + b)(a - b)$$

🎲 24. EXAMPLE

$$(2s + 7)(2s - 7)$$

Use:

$$(a + b)(a - b) = a^2 - b^2$$

Here:

$$a = 2s$$

$$b = 7$$

Therefore:

$$= (2s)^2 - 7^2$$

$$= 4s^2 - 49$$

🇮🇳 25. SHRĪDHARĀCHĀRYA AND QUICK SQUARING

The chapter connects the identity:

$$a^2 = (a + b)(a - b) + b^2$$

with Śhrīdharāchārya, who is described in the textbook as having proposed this method around 750 CE.

Example:

Find 55^2

Take:

$$a = 55$$

$$b = 5$$

Then:

$$55^2 = (55 + 5)(55 - 5) + 5^2$$

$$= 60 \times 50 + 25$$

$$= 3000 + 25$$

$$= \mathbf{3025}$$

26. ALGEBRA TILES

The chapter introduces **algebra tiles** to visualise factorisation.

Typical tiles represent:

$$\blacksquare x^2$$

$$\blacksquare x$$

$$\blacksquare 1$$

For example:

$$x^2 + 7x + 12$$

can be arranged into a rectangle.

The $7x$ is split as:

$$3x + 4x$$

and 12 can be arranged as:

$$3 \times 4$$

Therefore the rectangle has dimensions:

$$x + 3$$

and:

$$x + 4$$

So:

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$



27. FACTORISING $x^2 + 7x + 12$

We need two numbers whose:

$$\text{Sum} = 7$$

$$\text{Product} = 12$$

Those numbers are:

$$3 \text{ and } 4$$

Therefore:

$$x^2 + 7x + 12$$

$$= (x + 3)(x + 4)$$

★ 28. GENERAL IDENTITY

The chapter generalises this pattern:

$$(x + a)(x + b)$$

Expand:

$$= x^2 + bx + ax + ab$$

Therefore:

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

★ 29. ANOTHER IMPORTANT IDENTITY

For:

$$(ax + b)(cx + d)$$

multiply:

$$ax \times cx = acx^2$$

$$ax \times d = adx$$

$$b \times cx = bcx$$

$$b \times d = bd$$

Therefore:

$$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$$



30. FACTORISATION WITHOUT ALGEBRA TILES

Consider:

$$x^2 + 7x + 12$$

We compare it with:

$$x^2 + (a + b)x + ab$$

Therefore:

$$a + b = 7$$

and:

$$ab = 12$$

Numbers satisfying both are:

$$3 \text{ and } 4$$

Therefore:

$$(x + 3)(x + 4)$$



31. FACTORISING $x^2 + 11x + 30$

We need:

$$a + b = 11$$

and:

$$ab = 30$$

Factors of 30:

$$1 \times 30$$

$$2 \times 15$$

$$3 \times 10$$

$$5 \times 6$$

Only:

$$5 + 6 = 11$$

Therefore:

$$x^2 + 11x + 30 = (x + 5)(x + 6)$$

— 32. FACTORISING WITH NEGATIVE TERMS

Consider:

$$x^2 - 5x + 6$$

We need:

$$a + b = -5$$

$$ab = 6$$

Numbers are:

$$-2 \text{ and } -3$$

Therefore:

$$x^2 - 5x + 6 = (x - 2)(x - 3)$$



33. FACTORISATION SIGN TRICK

For:

$$x^2 + bx + c$$

If c is positive:

The two numbers have **same signs**.

If b is positive:

Both are positive.

If b is negative:

Both are negative.

Example:

$$x^2 + 7x + 12$$

$$\rightarrow +3, +4$$

Example:

$$x^2 - 7x + 12$$

$$\rightarrow -3, -4$$



34. CUBE OF A SUM

The chapter derives:

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Pattern:

1, 3, 3, 1



35. EXAMPLE

$$(x + 2)^3$$

$$= x^3 + 3x^2(2) + 3x(2^2) + 2^3$$

$$= x^3 + 6x^2 + 12x + 8$$



36. CUBE OF A DIFFERENCE

Replace b by $-b$.

We get:

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Notice the signs:

$+ \quad - \quad + \quad -$

The textbook highlights that the signs alternate.



37. MEMORY TRICK FOR CUBES

$$(a + b)^3$$

$$a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3$$

$$a^3 - 3a^2b + 3ab^2 - b^3$$

Remember:

Powers of a decrease:

$$3 \rightarrow 2 \rightarrow 1 \rightarrow 0$$

Powers of b increase:

$$0 \rightarrow 1 \rightarrow 2 \rightarrow 3$$



38. DIFFERENCE OF TWO CUBES

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$



39. SUM OF TWO CUBES

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

These identities are particularly useful for factorisation.



40. EXAMPLE

Factor:

$$x^3 - 8$$

Since:

$$8 = 2^3$$

we have:

$$x^3 - 2^3$$

Using:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Therefore:

$$x^3 - 8 = (x - 2)(x^2 + 2x + 4)$$

41. IDENTITY INVOLVING THREE CUBES

One of the most important identities in this chapter is:

$$x^3 + y^3 + z^3 - 3xyz$$

It can be factorised as:

$$(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

42. SPECIAL CASE

If:

$$x + y + z = 0$$

then:

$$x^3 + y^3 + z^3 = 3xyz$$

because the first factor becomes zero in the identity.

Thus:

★ If $x + y + z = 0$,

$$x^3 + y^3 + z^3 = 3xyz$$

This is a very useful result.



43. USING THE THREE-CUBE IDENTITY

Suppose:

$$x + y + z = 10$$

$$xyz = 25$$

and:

$$x^2 + y^2 + z^2 = 38$$

First use:

$$(x + y + z)^2$$

$$= x^2 + y^2 + z^2 + 2(xy + yz + zx)$$

Therefore:

$$100 = 38 + 2(xy + yz + zx)$$

So:

$$xy + yz + zx = 31$$

Then the three-cube identity gives:

$$x^3 + y^3 + z^3 = 145$$

The textbook works through this application in detail.



44. GEOMETRICAL INTERPRETATION OF CUBE IDENTITY

A cube with side:

$$a + b$$

has volume:

$$(a + b)^3$$

It can be divided into:



One cube of volume a^3



One cube of volume b^3



Three cuboids of volume a^2b



Three cuboids of volume ab^2

Therefore:

$$(a + b)^3$$

$$= a^3 + 3a^2b + 3ab^2 + b^3$$

This gives a geometric interpretation of the cubic identity.

45. SIMPLIFYING RATIONAL ALGEBRAIC EXPRESSIONS

The chapter next applies factorisation to rational expressions.

The main idea is:

- 1 Factorise numerator
 - 2 Factorise denominator
 - 3 Cancel common factors
 - 4 State restrictions on the denominator
-



46. EXAMPLE

Consider:

$$(x^2 - 7x + 12) / (5x^2 + 5x - 100)$$

Factor numerator:

$$\begin{aligned} x^2 - 7x + 12 \\ = (x - 3)(x - 4) \end{aligned}$$

Factor denominator:

$$\begin{aligned} 5x^2 + 5x - 100 \\ = 5(x^2 + x - 20) \\ = 5(x - 4)(x + 5) \end{aligned}$$

Therefore:

$$[(x - 3)(x - 4)] / [5(x - 4)(x + 5)]$$

Cancel common factor $(x - 4)$:

$$= (x - 3) / [5(x + 5)]$$

provided the original denominator is not zero.



47. IMPORTANT RESTRICTION

When cancelling a factor, remember:

You can cancel a common factor only when it is not zero.

For example:

$(x - 4)$ can be cancelled only when:

$$x \neq 4$$

The textbook explicitly notes the denominator restriction when simplifying rational expressions.



48. ALGEBRA TILES – KEY IDEA

For quadratic expressions:

$$x^2 + bx + c$$

look for two numbers a and b such that:

$$a + b = \text{coefficient of } x$$

and:

$$ab = \text{constant term}$$

Then:

$$x^2 + (a+b)x + ab$$

$$= (x+a)(x+b)$$



49. FACTORISATION CHECK

After factorising, always multiply the factors back.

Example:

$$(x + 3)(x + 4)$$

$$= x^2 + 4x + 3x + 12$$

$$= x^2 + 7x + 12$$

✓ Correct.



50. COMPLETE IDENTITY FORMULA SHEET

★ Identity 1

$$(x + y)^2 = x^2 + 2xy + y^2$$

★ Identity 2

$$(x - y)^2 = x^2 - 2xy + y^2$$

★ Identity 3

$$(x + y + z)^2$$

$$= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

★ Identity 4

$$(x + y)(x - y) = x^2 - y^2$$

★ Identity 5

$$(x + a)(x + b)$$

$$= x^2 + (a+b)x + ab$$

★ Identity 6

$$(ax + b)(cx + d)$$

$$= acx^2 + (ad+bc)x + bd$$

★ Identity 7

$$x^3 - y^3$$

$$= (x-y)(x^2 + xy + y^2)$$

★ Identity 8

$$x^3 + y^3$$

$$= (x+y)(x^2 - xy + y^2)$$

★ Identity 9

$$(x+y)^3$$

$$= x^3 + 3x^2y + 3xy^2 + y^3$$

★ Identity 10

$$(x-y)^3$$

$$= x^3 - 3x^2y + 3xy^2 - y^3$$

★ Identity 11

$$x^3 + y^3 + z^3 - 3xyz$$

$$= (x+y+z)(x^2+y^2+z^2-xy-xz-yz)$$

These are the identities listed in the chapter summary.

51. IDENTITY MEMORY TABLE

Identity	What to remember
$(a+b)^2$	$a^2 + 2ab + b^2$
$(a-b)^2$	$a^2 - 2ab + b^2$
$(a+b)(a-b)$	$a^2 - b^2$
$(a+b+c)^2$	Squares + twice pairwise products
$(a+b)^3$	$a^3 + 3a^2b + 3ab^2 + b^3$
$(a-b)^3$	Signs alternate

$$a^3 - b^3 \quad (a-b)(a^2+ab+b^2)$$

$$a^3 + b^3 \quad (a+b)(a^2-ab+b^2)$$

52. HOW TO CHOOSE THE RIGHT IDENTITY

If you see:

$$(a+b)^2$$

👉 Use:

$$a^2 + 2ab + b^2$$

If you see:

$$(a-b)^2$$

👉 Use:

$$a^2 - 2ab + b^2$$

If you see:

$$(a+b)(a-b)$$

👉 Use:

$$a^2 - b^2$$

If you see:

$$a^3 - b^3$$

👉 Use:

$$(a-b)(a^2+ab+b^2)$$

If you see:

$$a^3+b^3$$

👉 Use:

$$(a+b)(a^2-ab+b^2)$$

If you see:

$$x^2 + bx + c$$

👉 Find two numbers whose:

$$\text{sum} = b$$

$$\text{product} = c$$



53. COMMON MISTAKES

✗ Mistake 1

Writing:

$$(a+b)^2 = a^2+b^2$$

✓ **Correct:**

$$(a+b)^2 = a^2+2ab+b^2$$

✗ Mistake 2

Writing:

$$(a-b)^2 = a^2 - b^2$$

✓ **Correct:**

$$(a-b)^2 = a^2 - 2ab + b^2$$

✗ **Mistake 3**

Confusing:

$$a^3 + b^3$$

with:

$$(a+b)^3$$

They are NOT the same.

✗ **Mistake 4**

Forgetting the middle terms in:

$$(a+b+c)^2$$

There are **three pairwise products**, each multiplied by 2:

$$2ab + 2bc + 2ca$$

✗ **Mistake 5**

Cancelling factors without considering restrictions.

If denominator contains:

$$x - 4$$

then:

54. EXAM-IMPORTANT TOPICS

- ★ Definition of algebraic identity
 - ★ Difference between equation and identity
 - ★ Geometrical interpretation of identities
 - ★ Expansion using $(a+b)^2$
 - ★ Expansion using $(a-b)^2$
 - ★ Factorisation using identities
 - ★ Difference of squares
 - ★ Square of three terms
 - ★ Algebra tiles
 - ★ Factorisation of quadratic expressions
 - ★ Factorisation using splitting of middle term
 - ★ Cube of a sum
 - ★ Cube of a difference
 - ★ Sum of cubes
 - ★ Difference of cubes
 - ★ Three-variable cube identity
 - ★ Simplification of rational expressions
 - ★ Application-based factorisation problems
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55. QUICK REVISION

♦ Identity

An equation true for **all values** of its variables.

♦ Perfect Square

$$a^2 + 2ab + b^2 = (a+b)^2$$

♦ Perfect Square

$$a^2 - 2ab + b^2 = (a-b)^2$$

♦ Difference of Squares

$$a^2 - b^2 = (a+b)(a-b)$$

♦ Sum of Cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

♦ Difference of Cubes

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

♦ Cube of Sum

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

♦ Cube of Difference

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

♦ Three-Cube Identity

$$a^3 + b^3 + c^3 - 3abc$$

$$= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

56. CHAPTER TAKEAWAY

Chapter 4 shows that algebra is not just about manipulating symbols. **Identities reveal patterns that are always true**, and the chapter uses geometry, algebra tiles and factorisation to make those patterns visible.

The journey is:

 **Patterns**



 **Algebraic Identities**



 **Geometrical Visualisation**



 **Expansion**



 **Factorisation**



 **Algebra Tiles**



 **Cubic Identities**



 **Simplifying Rational Expressions**

 **Most important idea:**

An identity is a mathematical pattern that remains true no matter what values are substituted for its variables.