📘 CLASS 9 MATHEMATICS – CHAPTER 8 NOTES

🔢 Predicting What Comes Next: Exploring Sequences and Progressions

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Chapter 8 is "Predicting What Comes Next: Exploring Sequences and Progressions." It covers sequences, explicit and recursive rules, Arithmetic Progressions (AP), Geometric Progressions (GP), fractals and the Tower of Hanoi.

The new 2026–27 syllabus specifically includes **introduction to sequences, explicit/general rules, recursive rules, AP, nth term of AP, sum of first n natural numbers, GP, nth term of GP, fractals and Tower of Hanoi**.

🌟 CHAPTER 8: PREDICTING WHAT COMES NEXT

🎯 Learning Objectives

After studying this chapter, you should be able to:

✅ Understand sequences
✅ Identify patterns in sequences
✅ Predict subsequent terms
✅ Write an explicit/general rule
✅ Write a recursive rule
✅ Find terms using a given rule
✅ Identify Arithmetic Progressions
✅ Find the nth term of an AP
✅ Visualise APs using graphs
✅ Solve practical AP problems
✅ Find the sum of the first n natural numbers

✅ Identify Geometric Progressions
✅ Find the nth term of a GP
✅ Understand GP through visual patterns
✅ Apply GP to fractals
✅ Understand the Tower of Hanoi puzzle

📚 CHAPTER AT A GLANCE

8.1 🔢 Introduction to Sequences

8.2 🧠 Explicit or General Rule

8.3 🔄 Recursive Rule

8.4 ➕ Arithmetic Progressions

8.5 🧮 Sum of First n Natural Numbers

8.6 ✖️ Geometric Progressions

8.7 🌀 Fractals

8.8 🗼 Tower of Hanoi

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1. 🔢 WHAT IS A SEQUENCE?

A **sequence** is an ordered list of numbers in which each number is called a **term**. A sequence can be finite or infinite.

Example:

```
2, 4, 6, 8, 10, ...
```

Here:

2 → 1st term

4 → 2nd term

6 → 3rd term

8 → 4th term

and so on.

---

2. ⭐ COMMON NUMBER SEQUENCES

The textbook revisits several familiar sequences.

🔢 Natural Numbers

```
1, 2, 3, 4, 5, 6, ...
```

🔢 Odd Numbers

```
1, 3, 5, 7, 9, 11, ...
```

🔺 Triangular Numbers

```
1, 3, 6, 10, 15, 21, ...
```

🟦 Square Numbers

```
1, 4, 9, 16, 25, 36, ...
```

---

3. 🔍 FINDING THE PATTERN

To understand a sequence, look for:

➕ Addition

`3, 7, 11, 15, ...`

Add 4 each time.

➖ Subtraction

`20, 17, 14, 11, ...`

Subtract 3 each time.

✖️ Multiplication

`2, 6, 18, 54, ...`

Multiply by 3 each time.

➗ Division

`64, 32, 16, 8, ...`

Divide by 2 each time.

🔄 Alternating pattern

`2, 5, 3, 6, 4, 7, ...`

The rule may alternate between two operations/patterns.

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4. 🧠 TERMS OF A SEQUENCE

Terms are generally written as:

$$t_1, t_2, t_3, ..., t$$

where:

n = position of the term.

For example:

5, 8, 11, 14, ...

Here:

$t_1 = 5$

$t_2 = 8$

$t_3 = 11$

$t_4 = 14$

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5. ⭐ EXPLICIT / GENERAL RULE

An **explicit rule** gives a formula directly for the nth term.

For example:

2, 5, 8, 11, ...

The nth term is:

$$t = 3n - 1$$

Let's check:

For $n = 1$:

$3(1)-1 = 2$

For $n = 2$:

$3(2)-1 = 5$

For $n = 3$:

$3(3)-1 = 8$

✅ Correct.

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6. 🔎 WHY IS AN EXPLICIT RULE USEFUL?

Suppose we want the:

100th term

Instead of writing all 99 previous terms, we directly substitute:

$n = 100$

For:

$t \ = 3n - 1$

we get:

$3(100)-1$

$\mathbf{= 299}$

So:

⭐ $\mathbf{t_{100} = 299}$

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7. 🔄 RECURSIVE RULE

A **recursive rule** describes a term using one or more previous terms.

For example:

$2, 5, 8, 11, 14, ...$

We can write:

$$t_1 = 2$$

and:

$$t_n = t_{n-1} + 3$$

for $n \geq 2$.

This means:

Start with 2 and add 3 to get the next term.

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8. 🆚 EXPLICIT VS RECURSIVE

Explicit Rule	Recursive Rule
Gives term directly	Uses previous term(s)
Uses position n	Depends on earlier terms
Good for finding distant terms	Good for generating sequence step-by-step
Example $t_n = 3n - 1$	Example $t_n = t_{n-1} + 3$

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9. 🧮 EXAMPLE

Sequence:

$4, 7, 10, 13, ...$

Explicit rule:

Difference = 3

Therefore:

$t = 4 + (n-1)3$

$= 4 + 3n - 3$

$\mathbf{t = 3n + 1}$

Recursive rule:

$t_1 = 4$

$t = t_{-1} + 3$

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10. 🔢 TRIANGULAR NUMBERS

The sequence:

1, 3, 6, 10, 15, 21, ...

is called the **triangular number sequence**.

Notice the differences:

$3 - 1 = 2$

$6 - 3 = 3$

$10 - 6 = 4$

$15 - 10 = 5$

So the differences are:

2, 3, 4, 5, ...

The textbook also explains triangular numbers as sums of natural numbers: 1, 1+2, 1+2+3, etc.

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11. 🟦 SQUARE NUMBERS AND ODD NUMBERS

Square numbers:

```
1, 4, 9, 16, 25, ...
```

Differences:

```
3, 5, 7, 9, ...
```

These are consecutive odd numbers.

Therefore:

1 = 1

4 = 1 + 3

9 = 1 + 3 + 5

16 = 1 + 3 + 5 + 7

So:

⭐ Every square number is the sum of the first n odd numbers.

The textbook illustrates this relationship geometrically.

➕ 12. ARITHMETIC PROGRESSION – AP

An **Arithmetic Progression (AP)** is a sequence in which the difference between consecutive terms is constant.

That constant is called the:

Common Difference

and is represented by:

d

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13. ⭐ EXAMPLES OF AP

Example 1

```
2, 5, 8, 11, 14, ...
```

Common difference:

```
d = 3
```

Example 2

```
10, 7, 4, 1, -2, ...
```

Common difference:

```
d = -3
```

Example 3

```
1/2, 5/2, 9/2, 13/2, ...
```

Common difference:

```
d = 2
```

---

14. ❌ NOT AN AP

Consider:

$2, 4, 8, 16, \ldots$

Differences:

$2, 4, 8$

They are not constant.

Therefore:

❌ **Not an AP.**

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15. ⭐ GENERAL FORM OF AN AP

An AP can be written as:

$$a, a+d, a+2d, a+3d, \ldots$$

where:

a = first term

d = common difference.

The nth term is:

$$t \quad = a + (n-1)d$$

This is the main AP formula in the new textbook.

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16. 🧮 DERIVING THE nth TERM OF AN AP

Consider:

$a, a+d, a+2d, a+3d, \ldots$

Term positions:

Term	Value
1st	a
2nd	$a+d$
3rd	$a+2d$
4th	$a+3d$
nth	$a+(n-1)d$

Therefore:

⭐ $t = a + (n-1)d$

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17. 🧮 EXAMPLE – FIND THE 20th TERM

AP:

$5, 8, 11, 14, ...$

Here:

$a = 5$

$d = 3$

$n = 20$

Use:

$t = a + (n-1)d$

$= 5 + (20-1)3$

$= 5 + 57$

= 62

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18. 🧮 EXAMPLE – FIND THE 15th TERM

AP:

`12, 9, 6, 3, ...`

Here:

`a = 12`

`d = -3`

`n = 15`

t_{15} = 12 + (15-1)(-3)

`= 12 - 42`

= -30

---

19. 🔍 FINDING THE COMMON DIFFERENCE

For an AP:

d = second term - first term

Example:

`7, 11, 15, 19, ...`

`d = 11 - 7`

d = 4

20. 📈 VISUALISING AN AP

The textbook shows that when the terms of an AP are plotted against their positions, the points lie on a **straight line**.

For example:

`1, 5, 9, 13, 17, ...`

Plot:

`(1,1)`

`(2,5)`

`(3,9)`

`(4,13)`

`(5,17)`

These points lie on a straight line.

⭐ Key idea:

AP → linear pattern → straight-line graph

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21. 🧠 AP IN REAL LIFE

Arithmetic progressions occur when something changes by a **constant amount**.

Examples:

💰 Saving ₹100 more every month

🏟️ Seats increasing by a fixed number in successive rows

🌳 Planting a fixed number of additional trees each row

📚 Reading a fixed number of extra pages every day

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22. 🧮 FINDING THE NUMBER OF TERMS

Suppose:

AP:

```
5, 9, 13, ..., 81
```

Here:

```
a = 5
```

```
d = 4
```

Last term:

```
l = 81
```

Use:

```
l = a + (n-1)d
```

Therefore:

```
81 = 5 + (n-1)4
```

```
76 = 4(n-1)
```

```
19 = n-1
```

`n = 20`

So there are:

20 terms

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23. 🔢 SUM OF FIRST n NATURAL NUMBERS

The first n natural numbers are:

`1 + 2 + 3 + 4 + ... + n`

Their sum is:

⭐ n(n+1)/2

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24. 🧮 EXAMPLE

Find:

`1 + 2 + 3 + ... + 100`

Use:

`n(n+1)/2`

`= 100(101)/2`

`= 50 × 101`

`= 5050`

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25. 🧠 WHY DOES THE SUM FORMULA WORK?

Write the sum forward:

`1 + 2 + 3 + ... + n`

Now reverse it:

```
n + (n-1) + (n-2) + ... + 1
```

Add corresponding terms:

```
(n+1) + (n+1) + (n+1) + ...
```

There are n such pairs.

Therefore:

```
2S = n(n+1)
```

So:

S = n(n+1)/2

---

26. ✖️ GEOMETRIC PROGRESSION – GP

A **Geometric Progression (GP)** is a sequence in which each term is obtained by multiplying the previous term by the same constant number.

That constant is called the:

Common Ratio

represented by:

r

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27. ⭐ EXAMPLES OF GP

Example 1

```
2, 6, 18, 54, ...
```

Common ratio:

```
r = 3
```

Example 2

```
5, 10, 20, 40, ...
```

Common ratio:

```
r = 2
```

Example 3

```
81, 27, 9, 3, ...
```

Common ratio:

```
r = 1/3
```

---

28. 🔍 FINDING COMMON RATIO

For a GP:

```
r = second term / first term
```

Example:

```
3, 12, 48, 192, ...
```

```
r = 12/3
```

```
r = 4
```

---

29. ⭐ GENERAL FORM OF A GP

A GP can be written as:

$a, ar, ar^2, ar^3, \ldots$

where:

a = first term

r = common ratio.

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30. ⭐ nth TERM OF A GP

The nth term is:

$$t \quad = ar^{n-1}$$

This formula is given and applied in the textbook.

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31. 🧮 EXAMPLE – GP nth TERM

GP:

$3, 6, 12, 24, \ldots$

Here:

$a = 3$

$r = 2$

Find the 8th term.

$t_8 = ar^7$

$= 3 \times 2^7$

$= 3 \times 128$

$= 384$

---

32. 🧮 ANOTHER GP EXAMPLE

GP:

`5, 15, 45, 135, ...`

Here:

$a = 5$

$r = 3$

Therefore:

$t = 5 \times 3^{n-1}$

For $n = 4$:

$t_4 = 5 \times 3^3$

$= 5 \times 27$

$= 135$

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33. 🆚 AP VS GP

AP	GP
Constant difference	Constant ratio
Add/subtract same number	Multiply/divide by same number

`a, a+d, a+2d,...`	`a, ar, ar²,...`
nth term `a+(n-1)d`	nth term ar^{n-1}
Linear growth	Multiplicative/exponential growth

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34. 🧠 EASY MEMORY TRICK

AP ➕

Think:

Add the same number

Example:

`3, 7, 11, 15`

GP ✖️

Think:

Multiply by the same number

Example:

`3, 12, 48, 192`

---

35. 📈 AP AND GP GROWTH

AP

`2, 4, 6, 8, 10, ...`

Growth is by a fixed amount.

GP

```
2, 4, 8, 16, 32, ...
```

Growth happens by repeated multiplication.

Therefore GP can grow much faster.

🌀 36. FRACTALS

The chapter connects **Geometric Progressions** with **fractals**.

A fractal is a pattern that can show similar structures at different scales.

The textbook discusses the **Sierpiński triangle** as an example of a fractal.

🔺 37. SIERPIŃSKI TRIANGLE

Start with an equilateral triangle.

Stage 0

🔺 One triangle.

Stage 1

Join the midpoints and remove the central triangle.

Stage 2

Repeat the same process on the remaining triangles.

Stage 3

Repeat again.

The process can continue indefinitely.

This creates the:

Sierpiński Triangle

🧮 38. FRACTALS AND GP

At every stage, the number of certain components can change by a constant ratio.

For example, if the number of triangles becomes:

```
1, 3, 9, 27, ...
```

then:

```
r = 3
```

This is a:

Geometric Progression

🗼 39. TOWER OF HANOI

The chapter also introduces the famous:

Tower of Hanoi puzzle

It consists of:

🔵 Three pegs

💿 A number of differently sized disks

The disks begin stacked on one peg from largest to smallest.

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40. 🧩 RULES OF TOWER OF HANOI

The objective is to move all disks from one peg to another.

Rules:

1️⃣

Move only one disk at a time.

2️⃣

Only the top disk of a stack can be moved.

3️⃣

A larger disk cannot be placed on top of a smaller disk.

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41. 🔢 NUMBER OF MOVES

For n disks, the minimum number of moves is:

$$2^n - 1$$

Examples

1 disk

$2^1 - 1 = 1$

2 disks

$2^2 - 1 = 3$

3 disks

$2^3 - 1 = 7$

4 disks

$2^4 - 1 = 15$

5 disks

$2^5 - 1 = 31$

---

42. 🧠 TOWER OF HANOI AS A SEQUENCE

Number of moves:

1, 3, 7, 15, 31, ...

Notice:

$1 = 2^1 - 1$

$3 = 2^2 - 1$

$7 = 2^3 - 1$

$15 = 2^4 - 1$

Therefore:

$$T = 2^n - 1$$

This is an example of using a rule to predict a pattern.

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43. 🔄 RECURSIVE RULE FOR TOWER OF HANOI

The number of moves satisfies:

$$T = 2T_{-1} + 1$$

with:

$$T_1 = 1$$

Check:

$T_2 = 2(1)+1 = 3$

$T_3 = 2(3)+1 = 7$

$T_4 = 2(7)+1 = 15$

Correct! ✅

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44. 🧠 EXPLICIT VS RECURSIVE – TOWER OF HANOI

Explicit:

$T = 2^n - 1$

Recursive:

$T_1 = 1$

$T = 2T_{-1} + 1$

Both describe the same sequence.

---

45. 🔢 SEQUENCE PATTERN – IMPORTANT EXAMPLE

Consider:

```
1, 5, 9, 13, 17, 21, ...
```

Differences:

```
4, 4, 4, 4, ...
```

Therefore it is an AP.

Here:

```
a = 1
```

```
d = 4
```

So:

```
t  = 1 + (n-1)4
```

```
= 1 + 4n - 4
```

`t  = 4n - 3`

The textbook derives this exact rule from a growing square pattern.

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46. 📊 VISUAL PATTERNS AND AP

For the sequence:

```
1, 5, 9, 13, 17, ...
```

the textbook associates:

Stage	Number of squares
1	1
2	5
3	9

4	13
5	17
n	4n - 3

The ordered pairs:

(1,1), (2,5), (3,9), (4,13), (5,17)

lie on a straight line.

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47. 🧮 FINDING AN AP FROM A PATTERN

Suppose a growing pattern has:

3, 7, 11, 15, 19, ...

Difference:

$d = 4$

First term:

$a = 3$

Therefore:

$t = 3 + (n-1)4$

$= 3 + 4n - 4$

$\mathbf{t = 4n - 1}$

---

48. 🧮 FINDING A GP FROM A PATTERN

Suppose:

$2, 6, 18, 54, \ldots$

Common ratio:

$r = 3$

First term:

$a = 2$

Therefore:

$t_n = 2 \times 3^{n-1}$

For the 6th term:

$t_6 = 2 \times 3^5$

$= 2 \times 243$

$= \mathbf{486}$

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49. 🔍 HOW TO IDENTIFY AP OR GP

Step 1️⃣

Find differences.

If they are constant:

⭐ **AP**

Step 2️⃣

If differences are not constant, check ratios.

If:

$term_2/term_1$

$= term_3/term_2$

$= term_4/term_3$

then:

⭐ **GP**

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50. ⚠️ COMMON MISTAKES

❌ Mistake 1

Calling every patterned sequence an AP.

✅ AP requires a constant difference.

❌ Mistake 2

Confusing difference with ratio.

AP:

`5, 8, 11, 14`

Difference = `3`

GP:

`5, 15, 45, 135`

Ratio = `3`

❌ Mistake 3

Writing AP nth term as:

`a + nd`

❌ Wrong

✅ **Correct:**

$a + (n-1)d$

❌ **Mistake 4**

Writing GP nth term as:

ar^n

❌ **Wrong**

✅ **Correct:**

ar^{n-1}

❌ **Mistake 5**

Forgetting that the first term corresponds to $n = 1$.

📋 51. COMPLETE FORMULA SHEET

🔢 Sequence

Terms:

$t_1, t_2, t_3, ..., t$

➕ AP

General form:

$a, a+d, a+2d, ...$

Common difference:

$d = t_2 - t_1$

nth term:

$$t \quad = a + (n-1)d$$

🔢 Last term of AP

$l = a + (n-1)d$

➕ Sum of first n natural numbers

$$1+2+3+...+n = n(n+1)/2$$

✖️ GP

General form:

$a, ar, ar^2, ar^3, ...$

Common ratio:

$r = t_2/t_1$

nth term:

$$t \quad = ar^{n-1}$$

🗼 Tower of Hanoi

Minimum moves:

$$T = 2^{n} - 1$$

Recursive rule:

$T_1 = 1$

$T = 2T_{-1} + 1$

📊 52. AP VS GP – FINAL REVISION TABLE

Feature	AP ➕	GP ✖️
Pattern	Add/subtract	Multiply/divide
Constant	Difference	Ratio
First term	a	a
Common value	d	r
General form	a, a+d, ...	a, ar, ...
nth term	a+(n-1)d	ar^{n-1}
Graph	Straight line	Generally curved/exponential pattern
Example	2,5,8,11	2,6,18,54

🎯 53. EXAM-IMPORTANT QUESTIONS

Sequences

1. Find the next three terms.
2. Find the pattern in a sequence.
3. Find the explicit rule.
4. Find the recursive rule.
5. Find a particular term.

AP

6. Check whether a sequence is an AP.
7. Find the common difference.
8. Find the nth term.
9. Find a particular term.
10. Find the number of terms.
11. Solve real-life AP problems.
12. Represent an AP graphically.

Natural Numbers

13. Find the sum of the first n natural numbers.
14. Find the smallest n for which the sum exceeds a given number.

GP

15. Check whether a sequence is a GP.
16. Find the common ratio.
17. Find the nth term.
18. Solve growth/decay problems.

Fractals

19. Identify the GP pattern in a fractal.
20. Determine the number of shapes at a particular stage.

Tower of Hanoi

21. Find the minimum number of moves for n disks.
22. Write the recursive rule.
23. Find the number of moves for 5, 10 or 20 disks.

🧠 54. ONE-PAGE QUICK REVISION

🔢 Sequence

Ordered list of numbers.

📝 Explicit Rule

Gives the nth term directly.

🔄 Recursive Rule

Uses previous term(s).

➕ AP

Constant difference.

`a, a+d, a+2d,...`

⭐ AP nth term

`a+(n-1)d`

🔢 Natural-number sum

`n(n+1)/2`

✖️ GP

Constant ratio.

$a, ar, ar^2, ...$

⭐ GP nth term

ar^{n-1}

🌀 Fractals

Self-similar patterns; GP can describe quantities changing from stage to stage.

🗼 Tower of Hanoi

Minimum moves:

$2^n - 1$

🌟 55. CHAPTER TAKEAWAY

Chapter 8 is about **recognising patterns and turning those patterns into mathematical rules**.

👀 **Observe a pattern**

⬇️

🔍 **Find the rule**

⬇️

📝 **Write an explicit or recursive formula**

⬇️

🔢 **Predict future terms**

⬇️

➕ **Identify AP**

⬇️

✖️ **Identify GP**

⬇️

🌀 **Apply patterns to fractals**

⬇️

🗼 **Apply recursive thinking to Tower of Hanoi**

The new textbook particularly connects **visual patterns with algebraic rules**. For example, the growing-square pattern `1, 5, 9, 13, ...` leads to the explicit rule `t  = 4n - 3`, while its graph forms a straight line.

💡 Remember this:

AP → constant difference ➕
GP → constant ratio ✖️
Explicit rule → jump directly to any term 🎯
Recursive rule → build the sequence step by step 🔄