



MIDPOINT FORMULA



The **midpoint** divides a line segment into **two equal parts**.

THE MIDPOINT FORMULA

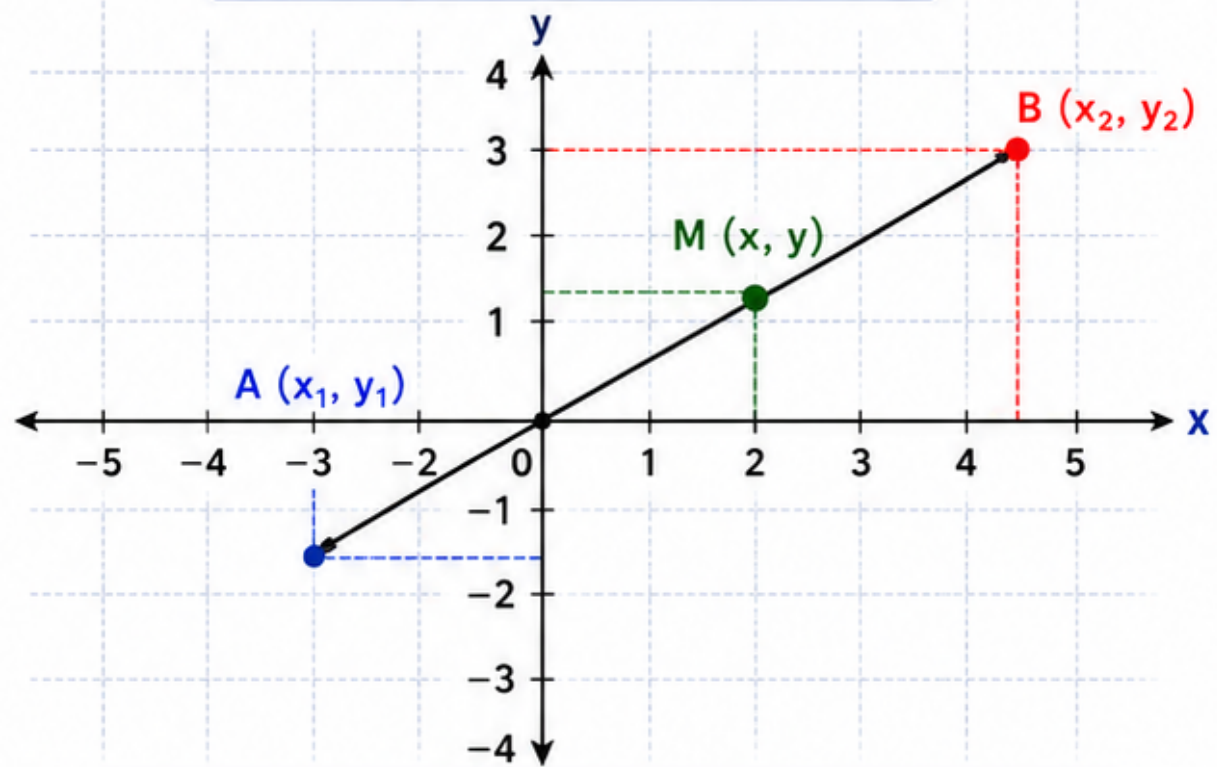
If **A** (x_1, y_1) and **B** (x_2, y_2) are the end points of a line segment, then the midpoint **M** (x, y) of $\overline{AB}$ is given by:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Where,

- (x_1, y_1) → coordinates of point **A**
- (x_2, y_2) → coordinates of point **B**
- (x, y) → coordinates of midpoint **M**

DIAGRAM ON COORDINATE PLANE



The midpoint **M** lies exactly halfway between **A** and **B** in both horizontal and vertical directions.

EXAMPLE 1

Find the midpoint of the line segment joining **A** ($-2, 4$) and **B** ($6, 0$).

Here, $x_1 = -2$, $y_1 = 4$, $x_2 = 6$, $y_2 = 0$

$$\begin{aligned} M(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-2 + 6}{2}, \frac{4 + 0}{2} \right) \\ &= \left(\frac{4}{2}, \frac{4}{2} \right) \\ &= (2, 2) \end{aligned}$$

EXAMPLE 2

Find the midpoint of the line segment joining **A** ($3, -1$) and **B** ($9, 7$).

Here, $x_1 = 3$, $y_1 = -1$, $x_2 = 9$, $y_2 = 7$

$$\begin{aligned} M(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{3 + 9}{2}, \frac{-1 + 7}{2} \right) \\ &= \left(\frac{12}{2}, \frac{6}{2} \right) \\ &= (6, 3) \end{aligned}$$

EXAMPLE 3

Find the midpoint of the line segment joining **A** ($-5, -2$) and **B** ($1, 6$).

Here, $x_1 = -5$, $y_1 = -2$, $x_2 = 1$, $y_2 = 6$

$$\begin{aligned} M(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-5 + 1}{2}, \frac{-2 + 6}{2} \right) \\ &= \left(\frac{-4}{2}, \frac{4}{2} \right) \\ &= (-2, 2) \end{aligned}$$

KEY POINTS

- ★ 1. The midpoint is the average of the **x-coordinates** and the average of the **y-coordinates**.
- ★ 2. The midpoint always lies **on the line** segment joining the two points.
- ★ 3. If **A** and **B** are the same point, then the midpoint is that point itself.
- ★ 4. The midpoint formula is very useful in geometry, coordinate geometry and real-life applications (e.g., finding the center of a line, center of mass, etc.).



SPECIAL CASES

Condition	Midpoint
If the line segment is horizontal ($y_1 = y_2$)	$M \left(\frac{x_1 + x_2}{2}, y_1 \right)$
If the line segment is vertical ($x_1 = x_2$)	$M \left(x_1, \frac{y_1 + y_2}{2} \right)$
From origin $(0, 0)$ to $P(x, y)$	$M \left(\frac{x}{2}, \frac{y}{2} \right)$
From A (x_1, y_1) to its reflection in x-axis A' ($x_1, -y_1$)	$M(x_1, 0)$
From A (x_1, y_1) to its reflection in y-axis A' ($-x_1, y_1$)	$M(0, y_1)$



Practice the **midpoint formula** and master **coordinate geometry**!





DISTANCE FORMULA



The distance between two points is the length of the line segment joining them.

THE DISTANCE FORMULA

If $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points in a coordinate plane, then the distance between them is

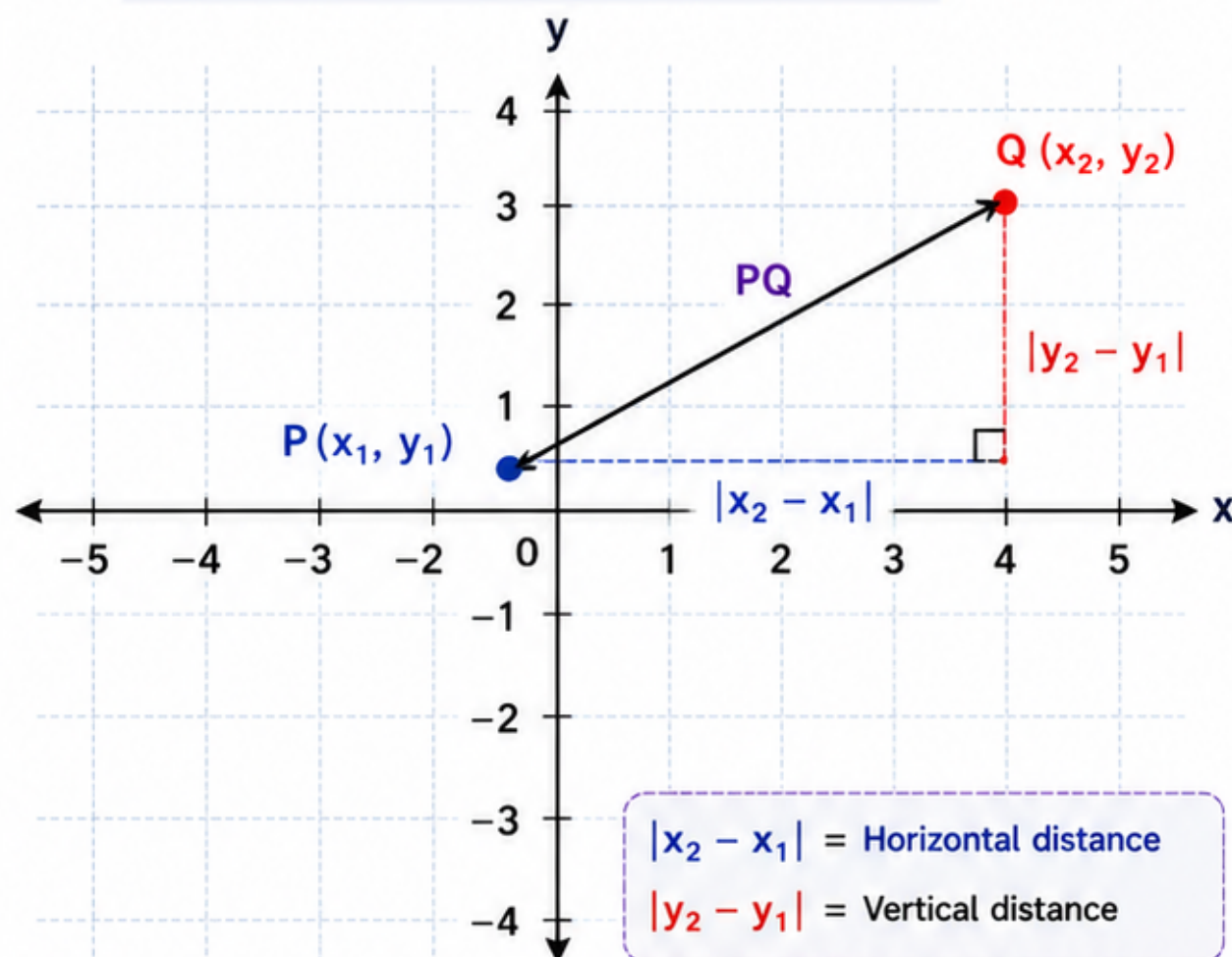
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Where,

$x_1, y_1 \rightarrow$ coordinates of point P

$x_2, y_2 \rightarrow$ coordinates of point Q

DIAGRAM ON COORDINATE PLANE



USING PYTHAGORAS THEOREM

Draw a right triangle using the horizontal and vertical lines.

$$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

EXAMPLE 1

Find the distance between $P(-2, 3)$ and $Q(4, 1)$.

Here, $x_1 = -2$, $y_1 = 3$, $x_2 = 4$, $y_2 = 1$

$$\begin{aligned} PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - (-2))^2 + (1 - 3)^2} \\ &= \sqrt{(6)^2 + (-2)^2} \\ &= \sqrt{36 + 4} \\ &= \sqrt{40} = 2\sqrt{10} \text{ units} \end{aligned}$$

EXAMPLE 2

Find the distance between $P(1, -5)$ and $Q(-3, 2)$.

Here, $x_1 = 1$, $y_1 = -5$, $x_2 = -3$, $y_2 = 2$

$$\begin{aligned} PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3 - 1)^2 + (2 - (-5))^2} \\ &= \sqrt{(-4)^2 + (7)^2} \\ &= \sqrt{16 + 49} \\ &= \sqrt{65} \text{ units} \end{aligned}$$

SPECIAL CASES

◆ Horizontal line ($y_1 = y_2$)

$$\text{Distance} = |x_2 - x_1|$$

◆ Vertical line ($x_1 = x_2$)

$$\text{Distance} = |y_2 - y_1|$$

◆ Distance from origin $(0, 0)$ to $P(x, y)$

$$\text{Distance} = \sqrt{x^2 + y^2}$$

KEY POINTS

- ★ Distance is always a positive value.
- ★ It is the shortest length between two points.
- ★ Used in geometry, coordinate geometry, physics and real-life applications.



QUICK REFERENCE TABLE

Points	Distance	Condition
$P(x_1, y)$ and $Q(x_2, y)$	$ x_2 - x_1 $	Same y (horizontal)
$P(x, y_1)$ and $Q(x, y_2)$	$ y_2 - y_1 $	Same x (vertical)
$P(x_1, y_1)$ and $Q(x_2, y_2)$	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	General case



Practice more problems to become a master in distance calculations!





REFLECTION RULES ON THE COORDINATE PLANE

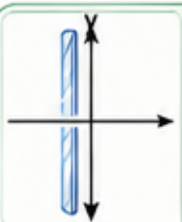


Reflection (or mirror image) of a point is **flipping** it over a line.
The distance from the point to the line **remains the same**.

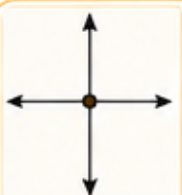
1. REFLECTION RULES



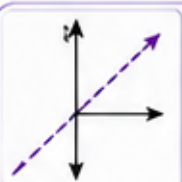
Reflection in the **x-axis**
 $(x, y) \rightarrow (x, -y)$
x remains the same,
y changes its sign.



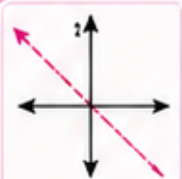
Reflection in the **y-axis**
 $(x, y) \rightarrow (-x, y)$
y remains the same,
x changes its sign.



Reflection in the origin (0, 0)
 $(x, y) \rightarrow (-x, -y)$
Both **x** and **y**
change their signs.



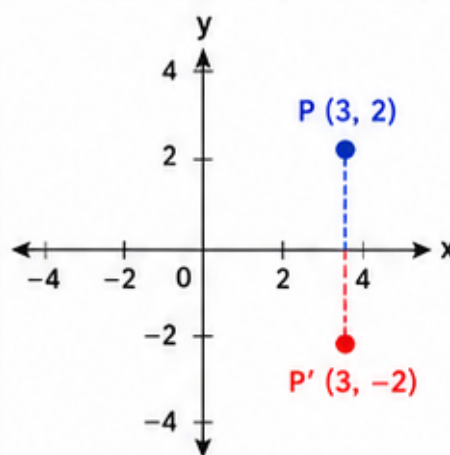
Reflection in the line **y = x**
 $(x, y) \rightarrow (y, x)$
x and **y** are interchanged.



Reflection in the line **y = -x**
 $(x, y) \rightarrow (-y, -x)$
x and **y** are interchanged
and signs are changed.

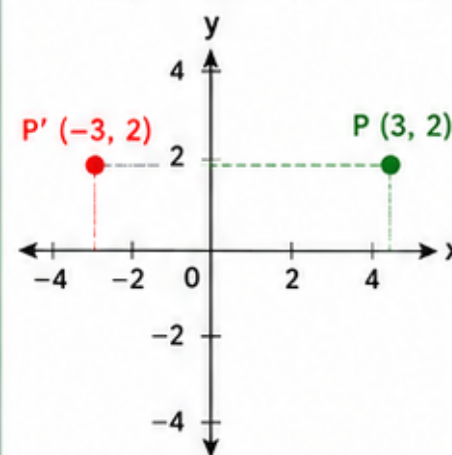
2. DIAGRAMS & EXAMPLES

In the **x-axis** : $(x, y) \rightarrow (x, -y)$



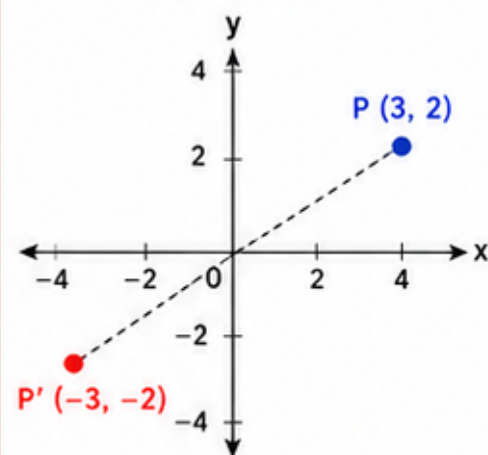
Example: $P(3, 2) \rightarrow P'(3, -2)$

In the **y-axis** : $(x, y) \rightarrow (-x, y)$



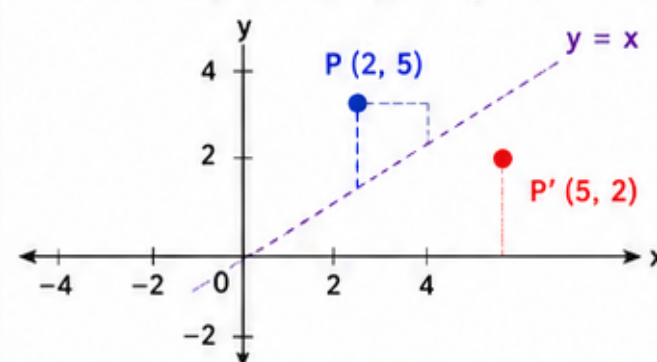
Example: $P(3, 2) \rightarrow P'(-3, 2)$

In the origin : $(x, y) \rightarrow (-x, -y)$



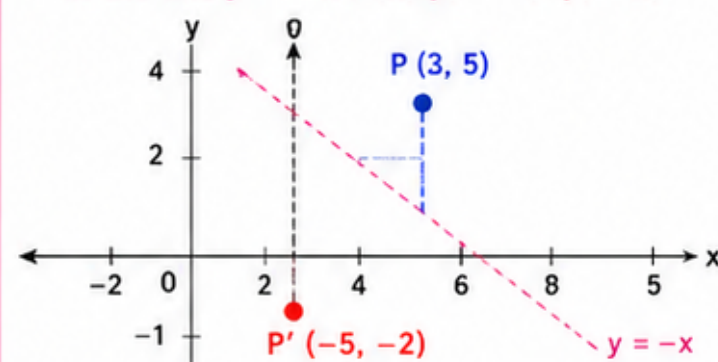
Example: $P(3, 2) \rightarrow P'(-3, -2)$

In the line **y = x** : $(x, y) \rightarrow (y, x)$



Example: $P(2, 5) \rightarrow P'(5, 2)$

In the line **y = -x** : $(x, y) \rightarrow (-y, -x)$



Example: $P(2, 5) \rightarrow P'(-5, -2)$

3. RULES SUMMARY TABLE

Line of Reflection	Rule	What Happens?
x-axis	$(x, y) \rightarrow (x, -y)$	x same, y changes sign
y-axis	$(x, y) \rightarrow (-x, y)$	y same, x changes sign
Origin (0, 0)	$(x, y) \rightarrow (-x, -y)$	Both change sign
y = x	$(x, y) \rightarrow (y, x)$	x and y are interchanged
y = -x	$(x, y) \rightarrow (-y, -x)$	x and y interchanged and signs changed

4. QUICK CHECK EXAMPLES

Point	In x-axis	In y-axis	In origin	In y = x	In y = -x
$P(x, y)$	$(x, -y)$	$(-x, y)$	$(-x, -y)$	(y, x)	$(-y, -x)$
(4, 3)	(4, -3)	(-4, 3)	(-4, -3)	(3, 4)	(-3, -4)
(-2, 5)	(-2, -5)	(2, 5)	(2, -5)	(5, -2)	(-5, 2)
(0, -7)	(0, 7)	(0, -7)	(0, 7)	(-7, 0)	(7, 0)

5. KEY POINTS TO REMEMBER

- ✓ Reflection does not change the distance of the point from the line.
- ✓ It only changes the position of the point to the opposite side.
- ✓ The mirror line acts as the perpendicular bisector of the segment joining the point and its image.
- ✓ Reflection is used in geometry, symmetry, graphs, patterns, and real-life applications (designs, logos, buildings, art, etc.).



6. REAL-LIFE APPLICATIONS

- ★ Symmetry in nature (butterfly wings, leaves, flowers)
- ★ Reflection in mirrors and water surfaces
- ★ Designing logos, rangoli, patterns, and artwork
- ★ Computer graphics and animations
- ★ Architecture and engineering (symmetric structures)



★ Understand the **rules**, **practice** more, and **master** reflections! ★



COORDINATE PLANE QUADRANT RULES

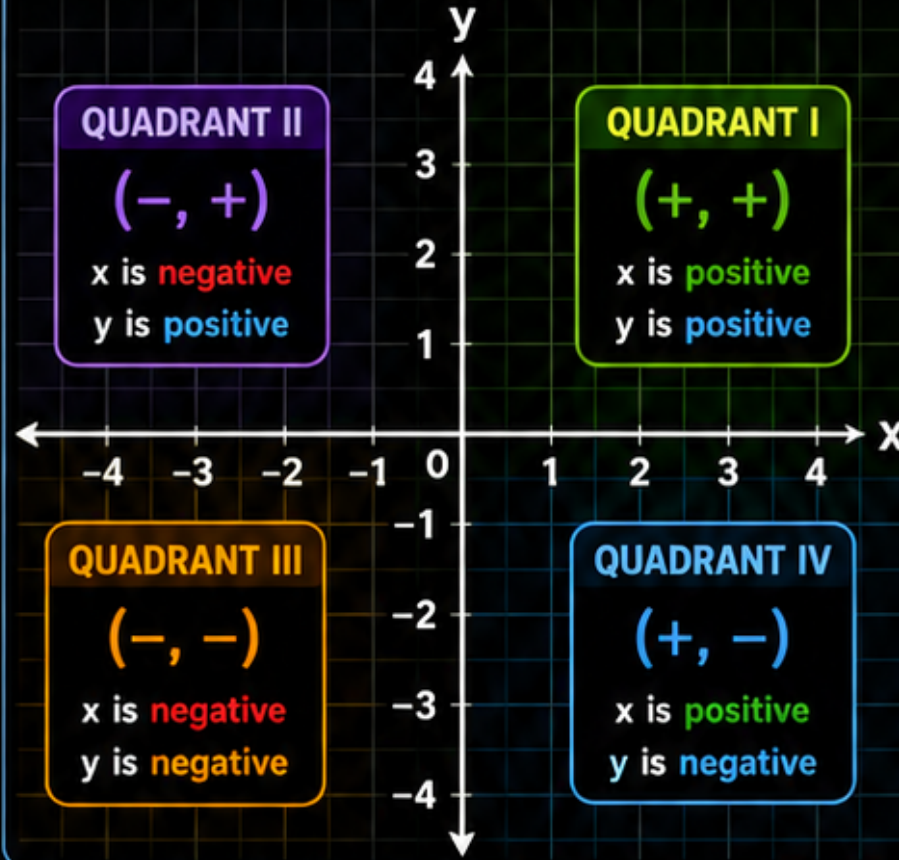


Understand the coordinate plane and the sign rules in all four quadrants!

1. THE COORDINATE PLANE

- ★ The coordinate plane is formed by two perpendicular number lines:
- ★ **X-axis** (horizontal)
→ Positive to the right
- ★ **Y-axis** (vertical)
→ Positive upward
- ★ The point where both axes intersect is the **Origin (0, 0)**.

2. THE FOUR QUADRANTS



3. SIGNS IN QUADRANTS

Quadrant	x	y	Sign Pair
Quadrant I (0 to $+\infty$, 0 to $+\infty$)	+	+	$(+, +)$
Quadrant II ($-\infty$ to 0, 0 to $+\infty$)	-	+	$(-, +)$
Quadrant III ($-\infty$ to 0, $-\infty$ to 0)	-	-	$(-, -)$
Quadrant IV (0 to $+\infty$, $-\infty$ to 0)	+	-	$(+, -)$

4. IMPORTANT POINTS ON AXES

- On the x-axis: $(x, 0)$
→ $y = 0$
- On the y-axis: $(0, y)$
→ $x = 0$
- At the origin: $(0, 0)$
→ Neither x nor y
(It is the intersection of both axes)

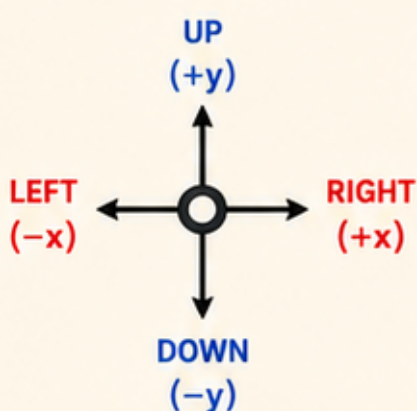
5. QUICK EXAMPLES

Point	Quadrant/ Axis	Sign of x	Sign of y
(3, 5)	Quadrant I	Positive	Positive
(-2, 4)	Quadrant II	Negative	Positive
(-3, -1)	Quadrant III	Negative	Negative
(4, -2)	Quadrant IV	Positive	Negative
(-5, 0)	On x-axis	Negative	Zero
(0, 7)	On y-axis	Zero	Positive
(0, 0)	Origin	Zero	Zero

6. KEY RULES TO REMEMBER

- ✓ Right → **x** increases **(+)**
- ✓ Left → **x** decreases **(-)**
- ✓ Up → **y** increases **(+)**
- ✓ Down → **y** decreases **(-)**
- ✓ Top half → **y** is positive **(+)**
- ✓ Bottom half → **y** is negative **(-)**
- ✓ Right half → **x** is positive **(+)**
- ✓ Left half → **x** is negative **(-)**
- ✓ The origin **(0, 0)** is neither positive nor negative.

7. PICTORIAL MEMORY



Quadrant II $(-, +)$	Quadrant I $(+, +)$
Quadrant III $(-, -)$	Quadrant IV $(+, -)$

8. REAL-LIFE APPLICATIONS

- Maps & GPS:** Locations are represented using coordinates.
- Games:** Position of characters or objects on a screen.
- Graphs & Charts:** Data plotting uses coordinate rules.
- Engineering & Architecture:** Designs and measurements.
- Physics:** Motion, velocity and displacement graphs.

★ Master the **quadrant rules**, and the **coordinate plane** becomes easy! ★