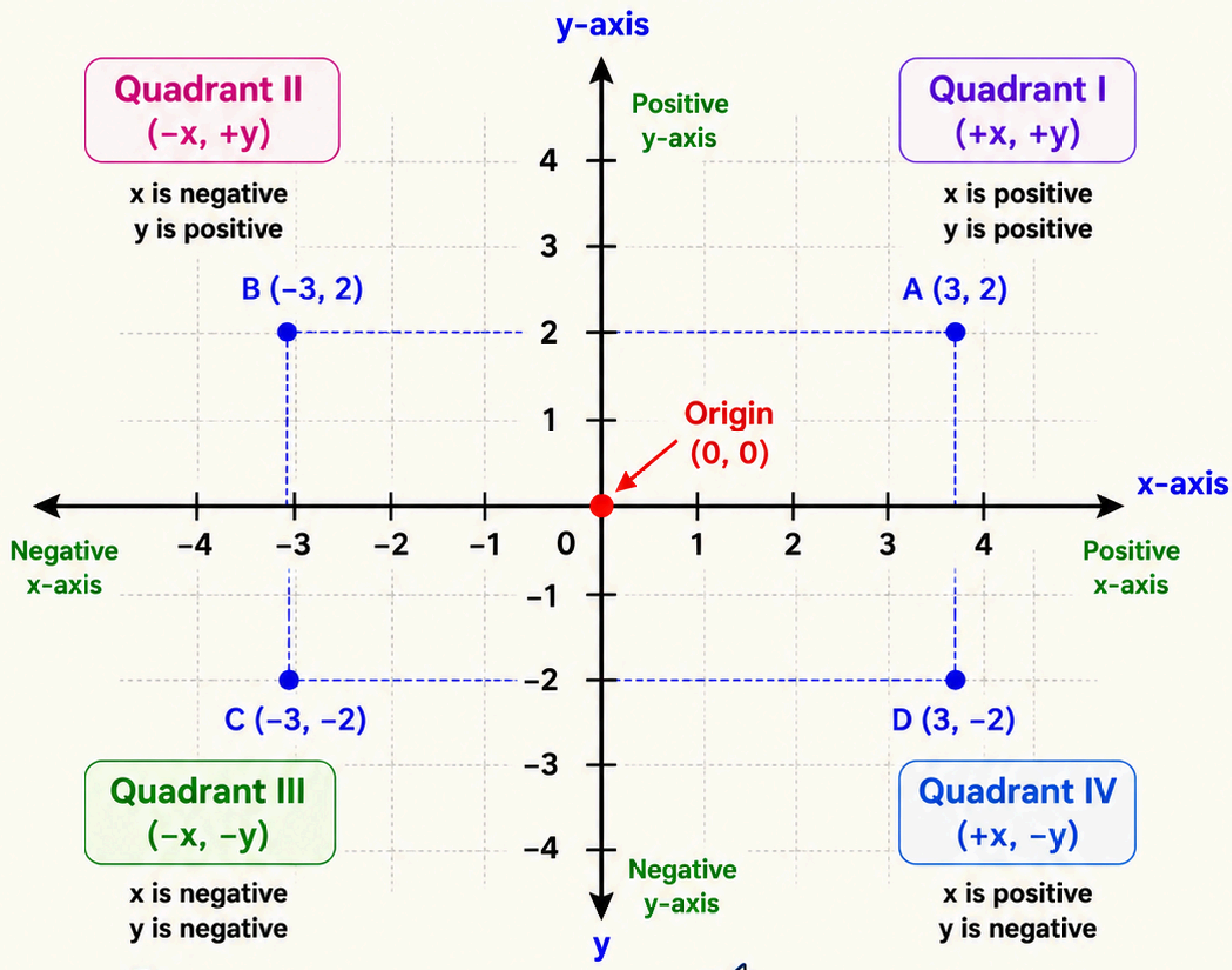
CARTESIAN COORDINATE PLANE

Axes, Origin, Quadrants and Coordinates



KEY POINTS

- The horizontal line is called x-axis.
- The vertical line is called y-axis.
- The point where they intersect is the **Origin (0, 0)**.
- A point in the plane is written as (x, y). Here, x is the x-coordinate and y is the y-coordinate.

EXAMPLE POINTS

A (3, 2)	→	x = 3, y = 2
B (−3, 2)	→	x = −3, y = 2
C (−3, −2)	→	x = −3, y = −2
D (3, −2)	→	x = 3, y = −2

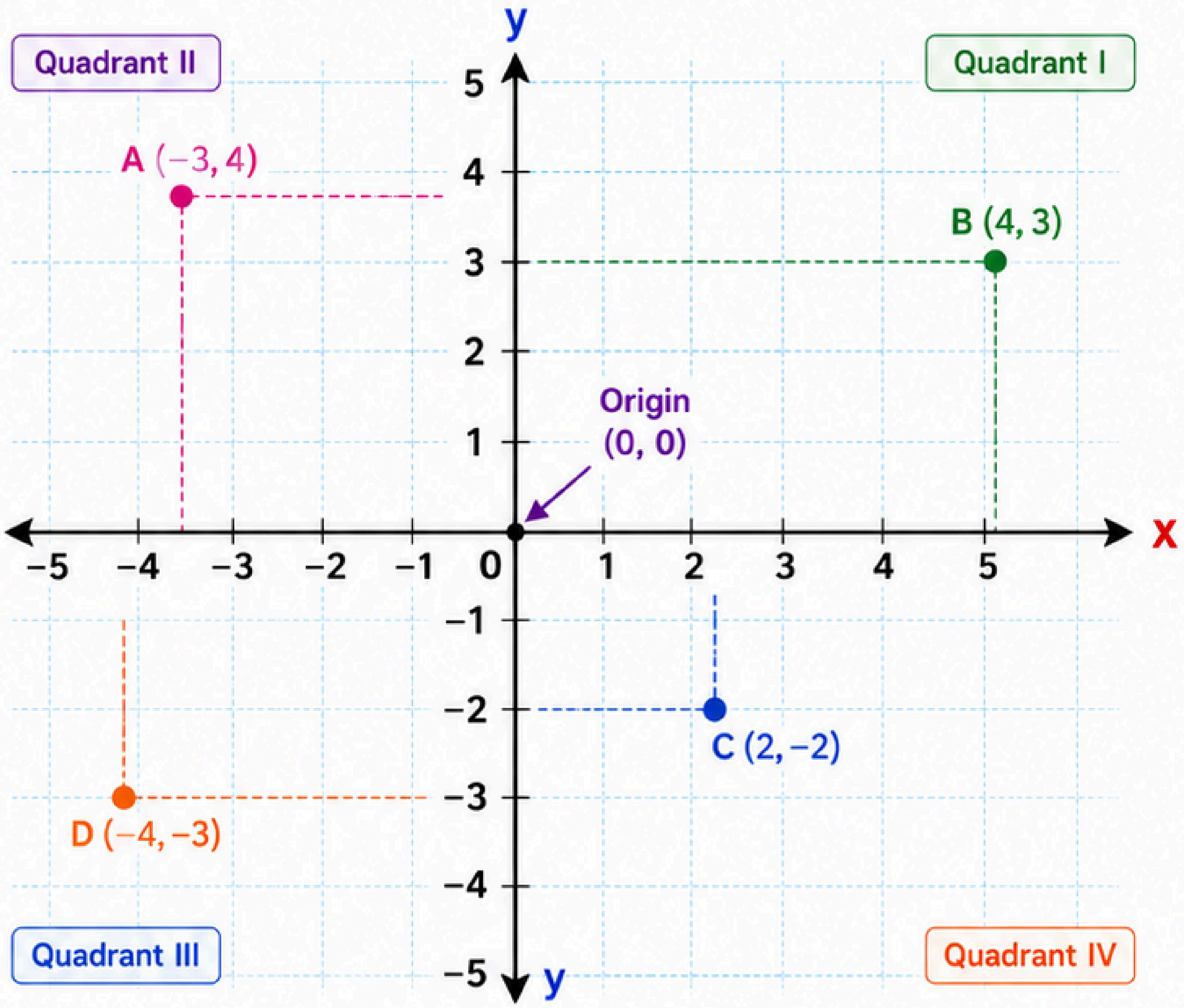
The coordinate plane helps us locate points, find distances, midpoints and study geometric shapes.

PLOTTING POINTS ON THE COORDINATE PLANE

A point in the plane is written as (x, y), where x is the horizontal distance from the origin along the x-axis and y is the vertical distance along the y-axis.

HOW TO PLOT A POINT (x, y)

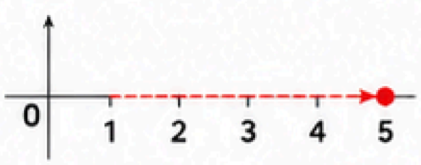
1. Start at the origin (0, 0).
2. Move $|x|$ units along the x-axis.
   - Right if x is positive.
   - Left if x is negative.
3. From that point, move $|y|$ units along the y-axis.
   - Up if y is positive.
   - Down if y is negative.
4. Mark the point and label it as (x, y).



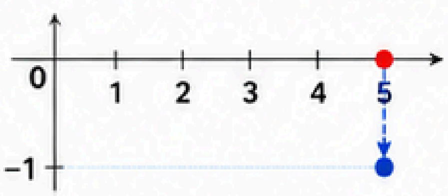
EXAMPLE 1

Plot the point P (5, −1).

Step 1:
From the origin (0, 0), move 5 units to the right along the x-axis.



Step 2:
From that point, move 1 unit down along the y-axis.

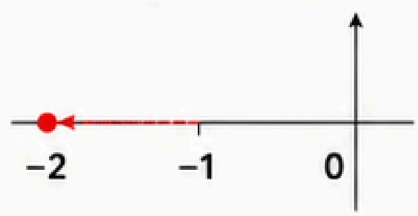


Hence, P is at (5, −1).

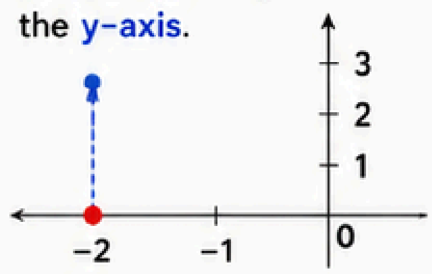
EXAMPLE 2

Plot the point Q (−2, 3).

Step 1:
From the origin (0, 0), move 2 units to the left along the x-axis.



Step 2:
From that point, move 3 units up along the y-axis.



Hence, Q is at (−2, 3).

QUICK NOTES

- The first value (x) is always read along the x-axis.
- The second value (y) is always read along the y-axis.
- Check the signs carefully to know the quadrant.

SAMPLE POINTS

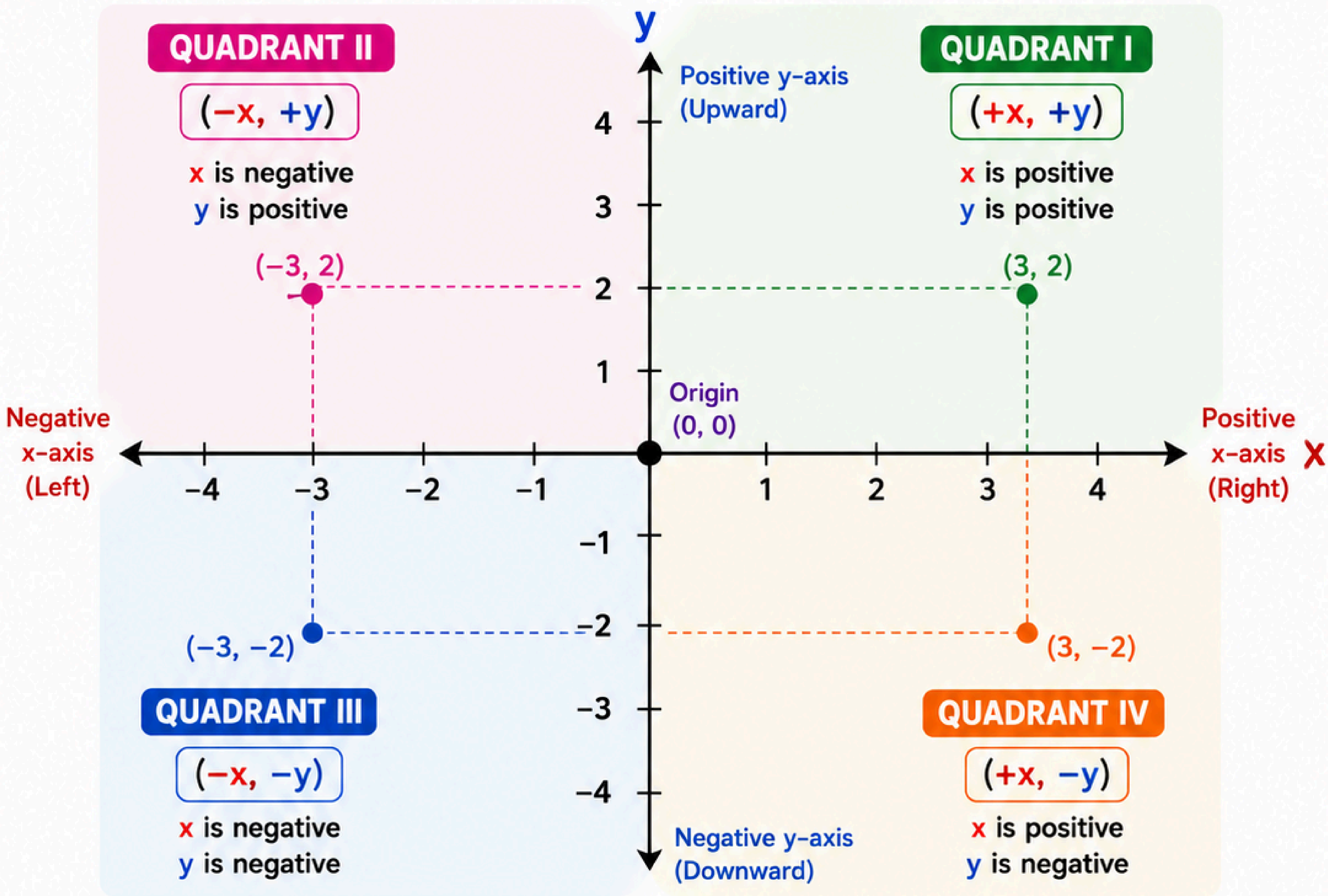
(3, 2)	→ Quadrant I	(−2, 5)	→ Quadrant II
(−4, −1)	→ Quadrant III	(1, −3)	→ Quadrant IV
(0, 4)	→ On y-axis	(−3, 0)	→ On x-axis

Practice plotting points to understand the coordinate plane better!

FOUR QUADRANTS & SIGNS

In the Coordinate Plane (Cartesian Plane)

The coordinate plane is divided into four regions called quadrants.
The signs of x and y coordinates in each quadrant are different.



QUADRANT	x-COORDINATE	y-COORDINATE
I First Quadrant	Positive (+)	Positive (+)
II Second Quadrant	Negative (−)	Positive (+)
III Third Quadrant	Negative (−)	Negative (−)
IV Fourth Quadrant	Positive (+)	Negative (−)

KEY POINTS

- A point in the plane is written as (x, y).
- The signs of x and y help us know in which quadrant a point lies.
- Points on the x-axis have y = 0.
- Points on the y-axis have x = 0.

Remember: Signs of coordinates decide the quadrant!

MIDPOINT OF A LINE SEGMENT

The midpoint divides a line segment into two equal parts.

MIDPOINT FORMULA

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are the end points of a line segment, then the midpoint $M(x, y)$ of $\overline{AB}$ is

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \ \frac{y_1 + y_2}{2} \right)$$

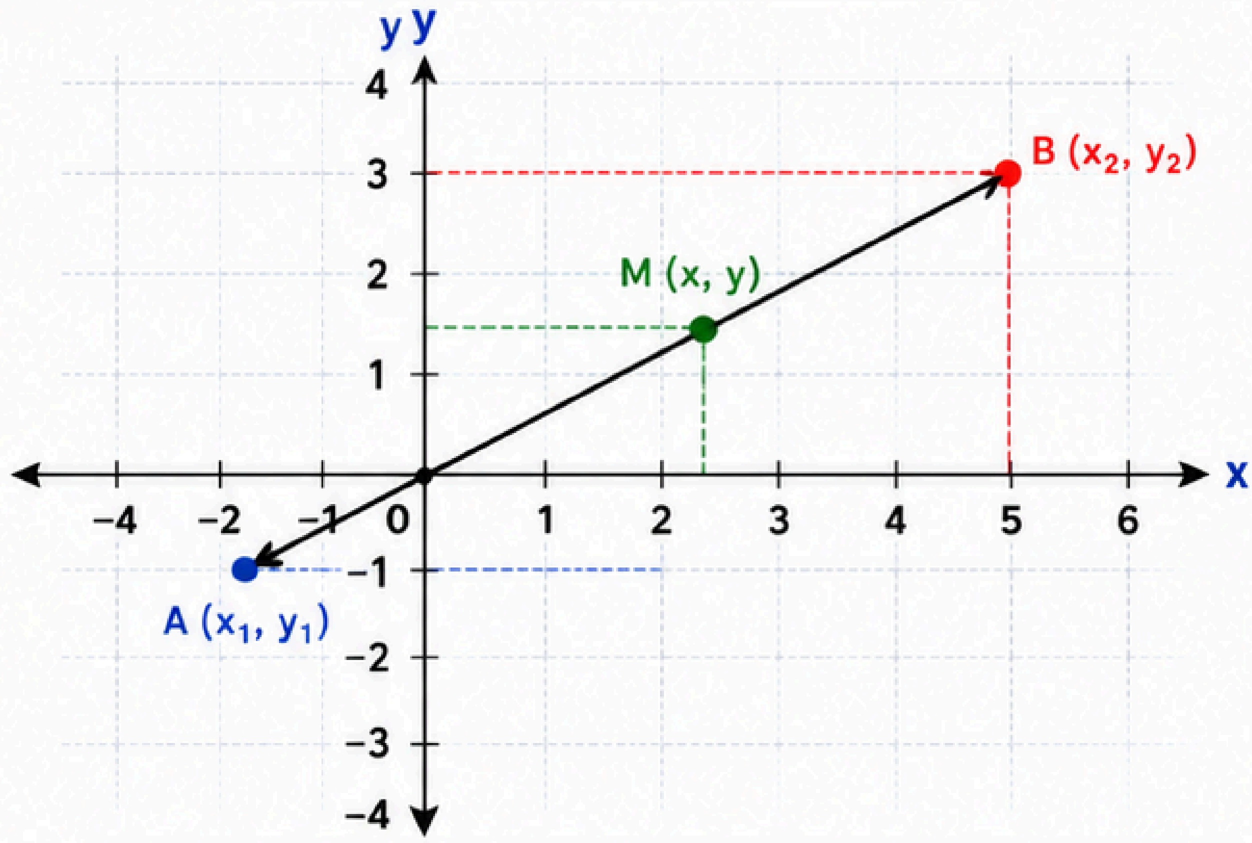
Where,

(x_1, y_1) → coordinates of point A

(x_2, y_2) → coordinates of point B

(x, y) → coordinates of midpoint M

DIAGRAM ON COORDINATE PLANE



The midpoint lies exactly halfway between the two end points in both horizontal and vertical directions.

EXAMPLE 1

Find the midpoint of the line segment joining A(−2, 3) and B(6, −1).

Here, $x_1 = -2$, $y_1 = 3$, $x_2 = 6$, $y_2 = -1$

$$\text{Midpoint } M(x, y) = \left(\frac{x_1 + x_2}{2}, \ \frac{y_1 + y_2}{2} \right)$$

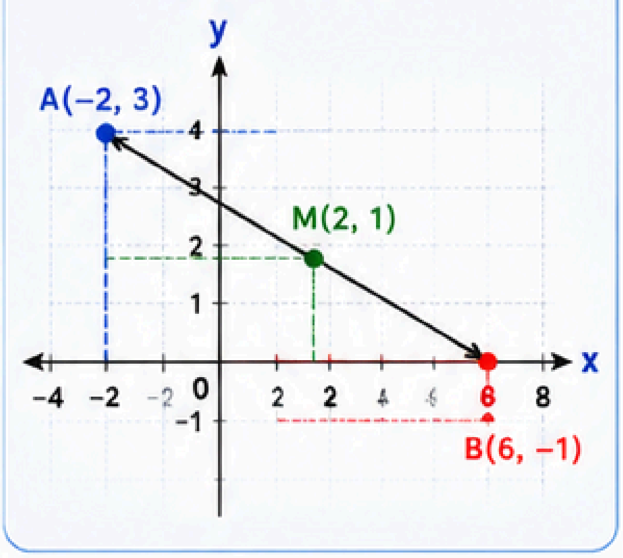
$$= \left(\frac{-2 + 6}{2}, \ \frac{3 + (-1)}{2} \right)$$

$$= \left(\frac{4}{2}, \ \frac{2}{2} \right)$$

$$= \boxed{(2, 1)}$$

VISUAL CHECK

For A(−2, 3) and B(6, −1), midpoint is (2, 1).



EXAMPLE 2

Find the midpoint of the line segment joining P(0, −4) and Q(8, 2).

Here, $x_1 = 0$, $y_1 = -4$, $x_2 = 8$, $y_2 = 2$

$$\text{Midpoint } M(x, y) = \left(\frac{x_1 + x_2}{2}, \ \frac{y_1 + y_2}{2} \right)$$

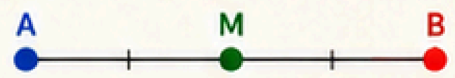
$$= \left(\frac{0 + 8}{2}, \ \frac{-4 + 2}{2} \right)$$

$$= \left(\frac{8}{2}, \ \frac{-2}{2} \right)$$

$$= \boxed{(4, -1)}$$

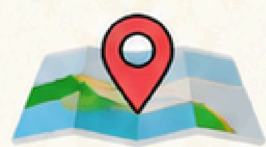
KEY POINTS

- The midpoint divides the line segment into two equal parts.



- The midpoint of $\overline{AB}$ is the average of the x-coordinates and the average of the y-coordinates of A and B.
- The midpoint always lies on the line segment joining the two points.

- If A and B are the same point, then the midpoint is that point itself.
- This formula is very useful in geometry, coordinate geometry and real-life problems.



(e.g., finding the center of a path, location between two places, etc.)

Practice using the midpoint formula to find the exact center between any two points!

DISTANCE BETWEEN TWO POINTS

The distance between two points is the length of the line segment joining them.

DISTANCE FORMULA

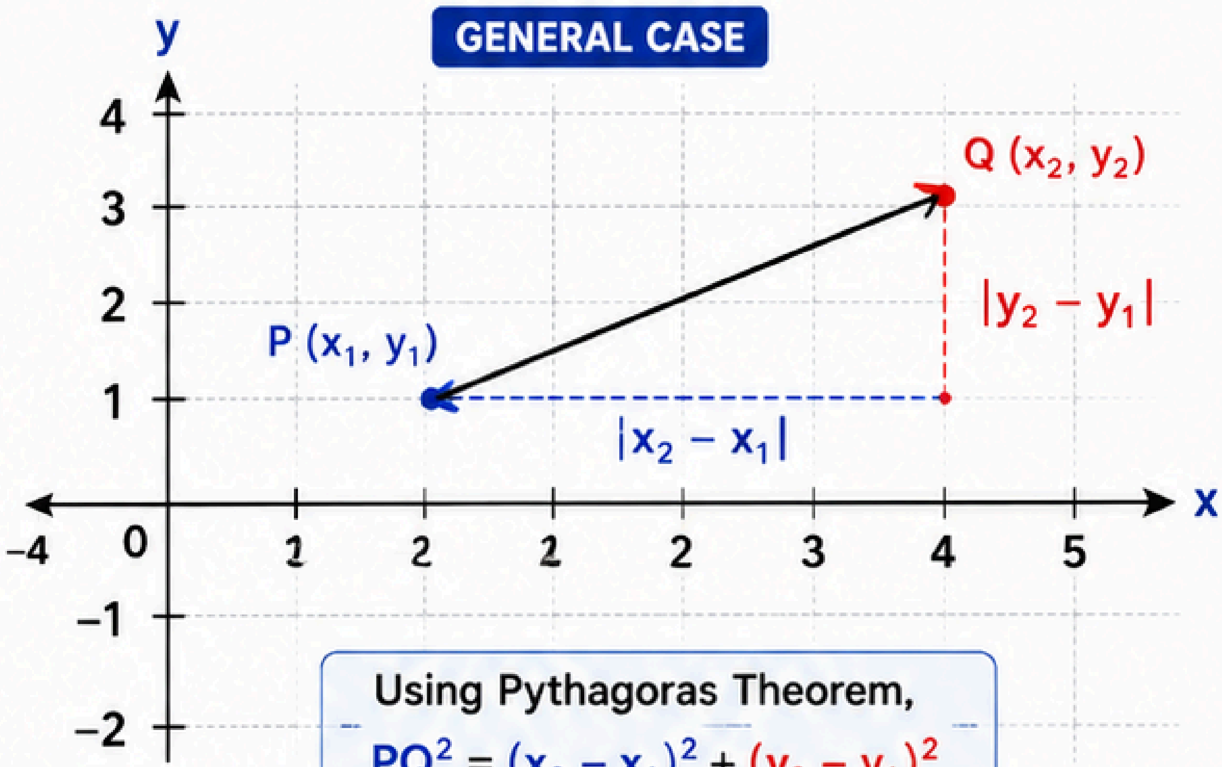
If $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points in the coordinate plane, then the distance between them is

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Where,

$x_1, y_1 \rightarrow$ coordinates of point P
$x_2, y_2 \rightarrow$ coordinates of point Q

GENERAL CASE

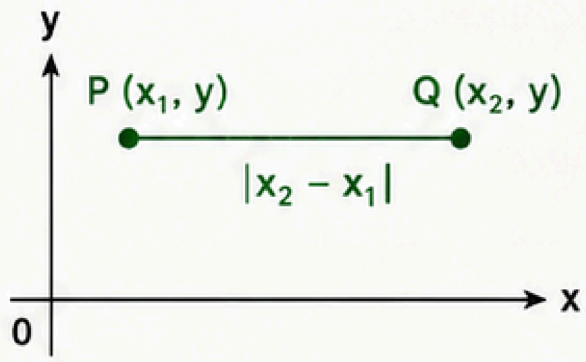


Using Pythagoras Theorem,
$$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

HORIZONTAL LINE

If $y_1 = y_2$, the line is horizontal.

Distance $= |x_2 - x_1|$



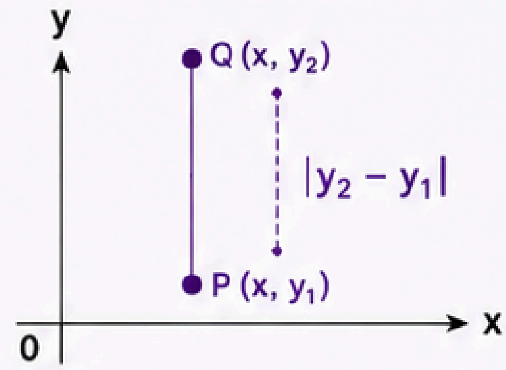
Example: P (−3, 2), Q (4, 2)

Distance $= |4 - (-3)| = |7| = 7$ units

VERTICAL LINE

If $x_1 = x_2$, the line is vertical.

Distance $= |y_2 - y_1|$



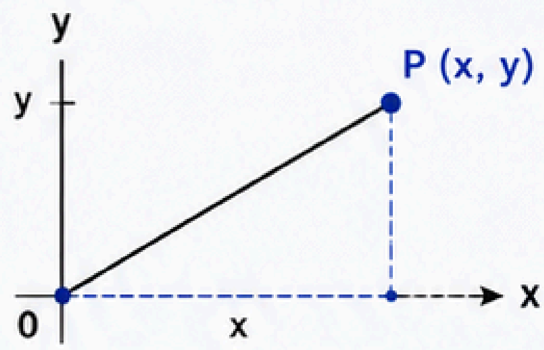
Example: P (3, −1), Q (3, 5)

Distance $= |5 - (-1)| = |6| = 6$ units

ORIGIN TO A POINT

Distance between (0, 0) and P(x, y)

$$= \sqrt{x^2 + y^2}$$



Example: P (−3, 4)

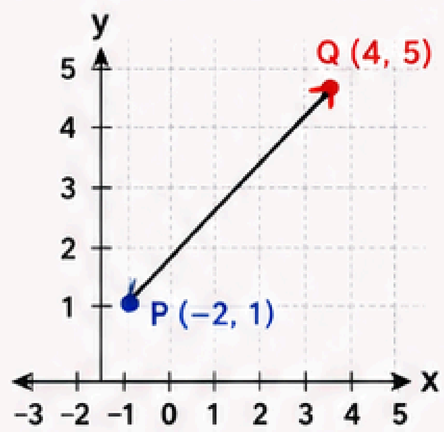
Distance $= \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16}$
$= \sqrt{25} = 5$ units

EXAMPLE (GENERAL CASE)

Find the distance between P (−2, 1) and Q (4, 5).

Here, $x_1 = -2$, $y_1 = 1$, $x_2 = 4$, $y_2 = 5$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4 - (-2))^2 + (5 - 1)^2}$$
$$= \sqrt{(6)^2 + (4)^2}$$
$$= \sqrt{36 + 16} = \sqrt{52} = 2\sqrt{13} \text{ units}$$

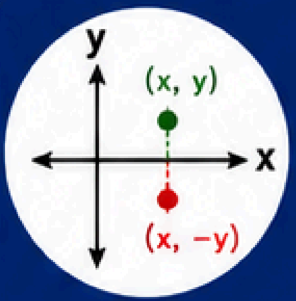


KEY POINTS

- The distance is always positive.
- It is the shortest length between two points.
- Used in many applications like geometry, physics and real life.
- Remember: Horizontal → use x values
  Vertical → use y values

Practice more to find distances quickly and accurately!

REFLECTION IN THE X-AXIS



When a point is reflected in the x-axis, the x-coordinate remains the same but the y-coordinate changes its sign.

RULE / FORMULA

If a point $P(x, y)$ is reflected in the x-axis, the image point $P'(x, -y)$.

$$P(x, y) \longrightarrow P'(x, -y)$$

That is,

- x remains the same
- y changes its sign

Example on diagram

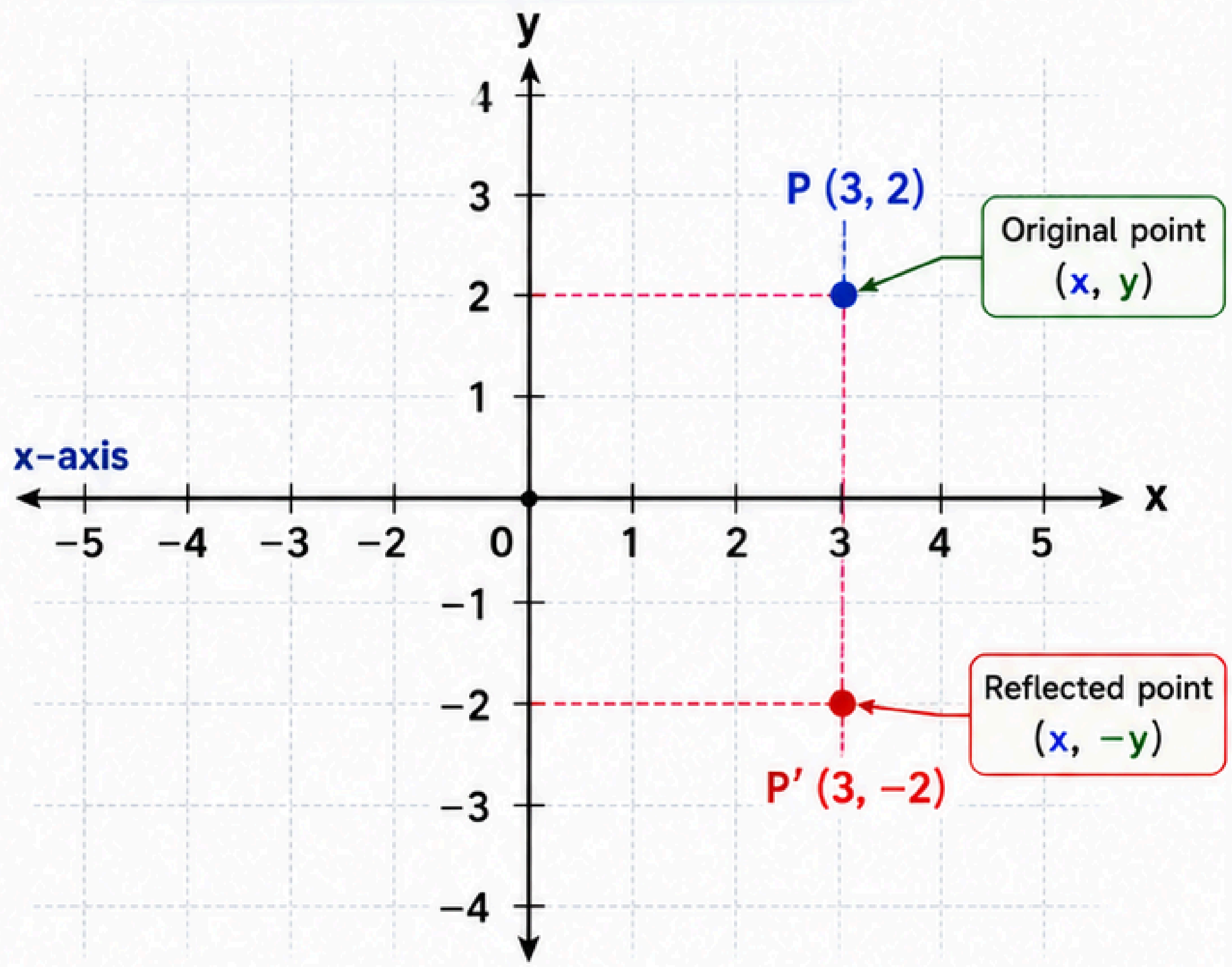
$P(3, 2)$ reflected in the x-axis

$\longrightarrow P'(3, -2)$

Here, $x = 3$ (same)

$y = 2$ changes to -2

DIAGRAM ON COORDINATE PLANE



MORE EXAMPLES

Original Point $P(x, y)$	Reflected Point in X-axis $P'(x, -y)$	Explanation
$(2, 5)$	$(2, -5)$	$x = 2$ same, $y = 5$ changes to -5
$(-4, 3)$	$(-4, -3)$	$x = -4$ same, $y = 3$ changes to -3
$(1, -6)$	$(1, 6)$	$x = 1$ same, $y = -6$ changes to 6
$(-3, -2)$	$(-3, 2)$	$x = -3$ same, $y = -2$ changes to 2
$(0, 7)$	$(0, -7)$	$x = 0$ same, $y = 7$ changes to -7

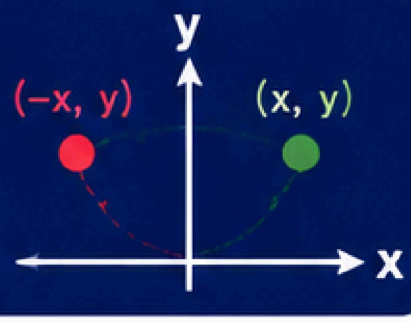
KEY POINTS

- Reflection in the x-axis keeps the x-coordinate unchanged.
- The y-coordinate changes its sign.
- Points on the x-axis do not change after reflection. (Example: $(2, 0) \longrightarrow (2, 0)$)
- Points above the x-axis ($y > 0$) are reflected below the x-axis ($y < 0$).
- Points below the x-axis ($y < 0$) are reflected above the x-axis ($y > 0$).
- The x-axis acts like a mirror.

Remember: (x, y) reflected in the x-axis becomes $(x, -y)$.

REFLECTION IN THE Y-AXIS

When a point is reflected in the y-axis, the y-coordinate remains the same but the x-coordinate changes its sign.



RULE / FORMULA

If a point P(x, y) is reflected in the y-axis, the image point is P′(−x, y).

$$\boxed{P(x, y) \longrightarrow P'(-x, y)}$$

That is,

- x changes its sign
- y remains the same

EXAMPLE ON DIAGRAM

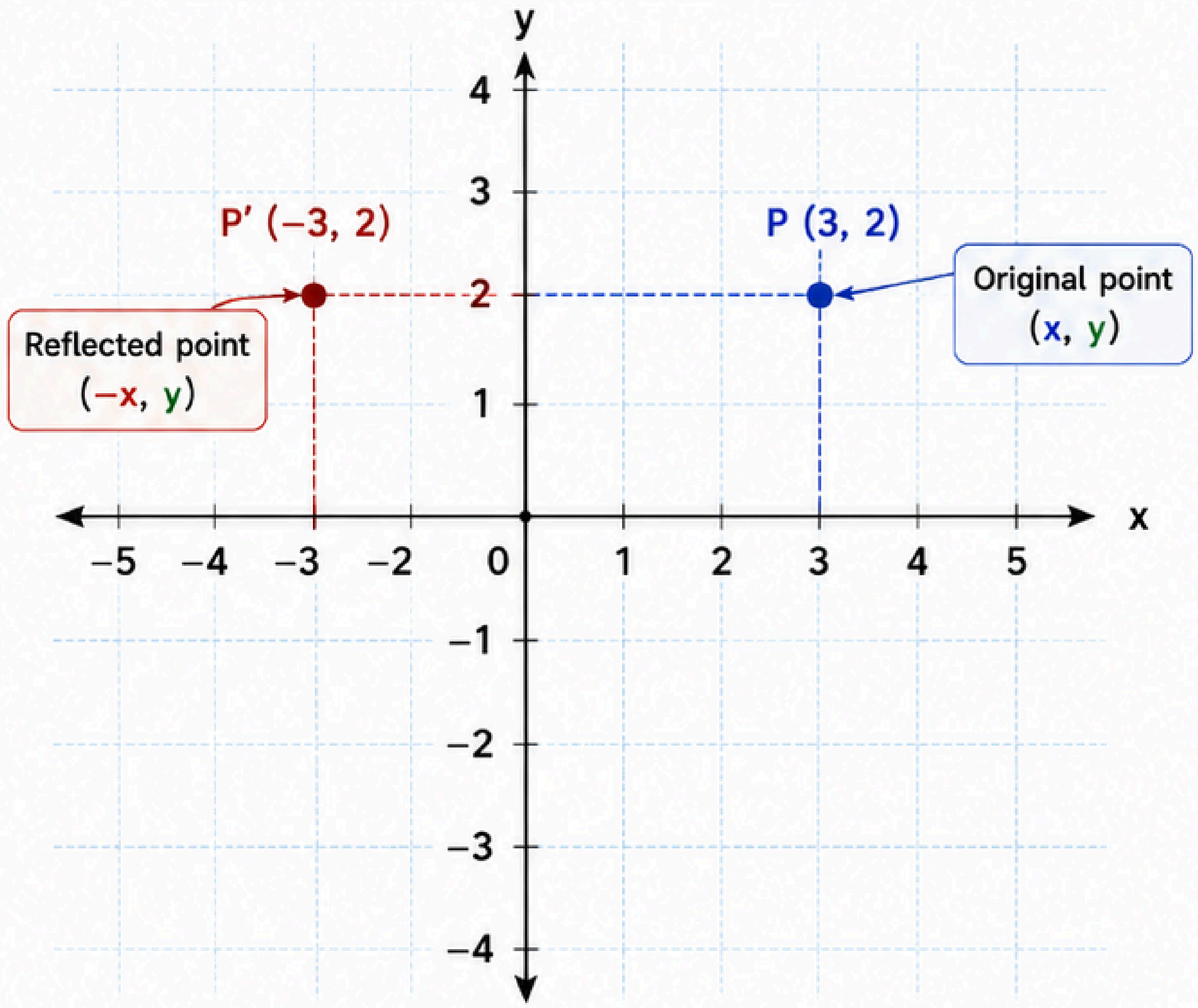
P(3, 2) is reflected in the y-axis

⟶ P′(−3, 2)

Here,

- x = 3 changes to −3
- y = 2 remains the same

DIAGRAM ON COORDINATE PLANE



MORE EXAMPLES

Original Point P(x, y)	Reflected Point in Y-axis P′(−x, y)	Explanation
(2, 5)	(−2, 5)	x = 2 changes to −2, y = 5 same
(−4, 3)	(4, 3)	x = −4 changes to 4, y = 3 same
(1, −6)	(−1, −6)	x = 1 changes to −1, y = −6 same
(−3, −2)	(3, −2)	x = −3 changes to 3, y = −2 same
(0, 7)	(0, 7)	x = 0 remains 0, y = 7 same

KEY POINTS

- ★ Reflection in the y-axis keeps the y-coordinate unchanged.
- ★ The x-coordinate changes its sign.
- ★ Points on the y-axis do not change after reflection. (Example: (0, 3) ⟶ (0, 3))
- ★ Points on the right side (x > 0) are reflected to the left side (x < 0) at the same distance from the y-axis.
- ★ Points on the left side (x < 0) are reflected to the right side (x > 0) at the same distance from the y-axis.
- ★ The y-axis acts like a mirror.

Remember: (x, y) reflected in the y-axis becomes (−x, y).