



Class 9 Maths – Chapter 5

I'm Up and Down, and Round and Round

New NCERT Ganita Manjari | Grade 9 | Part I

Chapter 5 is the **Circles** chapter. It develops the properties of circles, chords, arcs, angles, circumcircles and cyclic quadrilaterals, with an emphasis on reasoning and proofs.



CHAPTER 5: I'M UP AND DOWN, AND ROUND AND ROUND



Learning Objectives

By the end of this chapter, you should understand:

- Circles and their basic elements
 - Chords and arcs
 - Central angles
 - Angles subtended by chords and arcs
 - Properties of equal chords
 - Perpendicular from the centre to a chord
 - Distance of chords from the centre
 - Circumcircle and circumcentre
 - Angles subtended by an arc
 - Angle in a semicircle
 - Concyclic points
 - Cyclic quadrilaterals
 - Opposite angles of a cyclic quadrilateral
-

1. BASIC TERMS RELATED TO A CIRCLE

A **circle** is the set of all points in a plane that are at the same distance from a fixed point.

The fixed point is called the:

Centre

The constant distance from the centre to any point on the circle is the:

Radius

Important parts of a circle

♦ **Centre**

The fixed point from which every point on the circle is equally distant.

♦ **Radius**

A line segment joining the centre to any point on the circle.

♦ **Diameter**

A chord passing through the centre.

♦ **Chord**

A line segment whose endpoints lie on the circle.

♦ **Arc**

A part of the circumference of a circle.

2. CHORD

A **chord** joins any two points on a circle.

For example, if A and B lie on a circle, then:

AB is a chord.

A diameter is also a chord, but it is the **longest chord** of a circle.

3. DIAMETER

A diameter passes through the centre.

If the radius is r , then:

Diameter = $2r$

Therefore:

$$d = 2r$$

and

$$r = d/2$$

4. ARC

An **arc** is a portion of the circumference of a circle.

There are two types:

Minor arc

An arc corresponding to a central angle **less than 180°** .

Major arc

An arc corresponding to a central angle **greater than 180°** .

5. CIRCLES THROUGH POINTS

Through one point

Infinitely many circles can pass through a given point.

Through two points

Infinitely many circles can pass through two given points.

Their centres lie on the **perpendicular bisector** of the line segment joining the two points.

Through three non-collinear points

Exactly **one circle** can be drawn through three points that are not on the same straight line.

This circle is called the:

Circumcircle

of the triangle formed by the three points.

Its centre is called the:

Circumcentre

The circumcentre lies at the intersection of the perpendicular bisectors of the sides of the triangle.

6. ANGLE SUBTENDED BY A CHORD

A chord can form an angle at a point.

For example, chord AB subtends an angle at a point C on the circle.

The angle formed is:

$\angle ACB$

The chapter develops the relationship between angles subtended by the same arc at different points on the circle.

7. ★ EQUAL CHORDS AND CENTRAL ANGLES

One important theorem is:

Equal chords of a circle subtend equal angles at the centre.

Conversely:

Chords that subtend equal angles at the centre are equal in length.

Therefore:

If:

$$AB = CD$$

then:

$$\angle AOB = \angle COD$$

where O is the centre.

And conversely:

If:

$$\angle AOB = \angle COD$$

then:

$$AB = CD$$

8. ★ CENTRE AND MIDPOINT OF A CHORD

Theorem:

The line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.

Suppose:

- O = centre
- AB = chord
- M = midpoint of AB

Then:

$$AM = MB$$

and:

$$OM \perp AB$$

9. ↺ CONVERSE OF THE THEOREM

The converse is also true:

The perpendicular from the centre of a circle to a chord bisects the chord.

So if:

$$OM \perp AB$$

then:

$$AM = MB$$

This is an important theorem for problems involving chords.

10. 🧠 WHY DOES THIS WORK?

Suppose O is the centre and M is the midpoint of chord AB.

Since:

$$OA = OB$$

the triangle OAB is isosceles.

Also:

$$AM = BM$$

Using SAS congruence:

$$\triangle OMA \cong \triangle OMB$$

Therefore:

$$\angle OMA = \angle OMB$$

But they form a straight line:

$$\angle OMA + \angle OMB = 180^\circ$$

So each angle is:

$$90^\circ$$

Therefore:

$$OM \perp AB$$

11. ★ CHORDS AND DISTANCE FROM THE CENTRE

Another important result:

Equal chords lie at equal distances from the centre.

Conversely:

Chords at equal distances from the centre are equal in length.

12. LONGER CHORD – CLOSER TO CENTRE

For two unequal chords:

The longer chord is closer to the centre.

Therefore:

Longer chord → smaller distance from centre

Shorter chord → greater distance from centre

13. ANGLE SUBTENDED BY AN ARC

One of the most important results in this chapter is:

The angle subtended by an arc at the centre is twice the angle subtended by the same arc at any point on the remaining part of the circle.

If an arc AB subtends:

$\angle AOB$

at the centre and:

$\angle ACB$

at a point C on the circle, then:

$$\angle AOB = 2\angle ACB$$

or:

$$\angle ACB = \frac{1}{2}\angle AOB$$

14. EXAMPLE

Suppose:

$$\angle AOB = 100^\circ$$

Then the angle subtended by the same arc at C is:

$$\angle ACB = 100^\circ / 2$$

$$= 50^\circ$$

15. ANGLES SUBTENDED BY THE SAME ARC

If two points lie on the same part of a circle and see the same chord/arc, the angles subtended by that arc are equal.

This is an important idea used in many circle proofs.

16. ANGLE IN A SEMICIRCLE

A diameter subtends an angle of:

$$90^\circ$$

at any point on the circle.

This is commonly called:

Angle in a semicircle is a right angle.

Example

If AB is a diameter and C is any point on the circle, then:

$$\angle ACB = 90^\circ$$

This is one of the most important results to remember for examinations.

17. CONCYCLIC POINTS

Four or more points are called **concyctic** if they lie on the same circle.

For example:

A, B, C, D

are concyclic if one circle can pass through all four points.

18. A USEFUL CONVERSE

If a line segment joining two points subtends **equal angles at two other points on the same side**, then all four points lie on one circle.

Therefore, the four points are:

Concyctic

19. CYCLIC QUADRILATERAL

A quadrilateral whose four vertices lie on a circle is called a:

Cyclic Quadrilateral

For example:

ABCD

is cyclic if A, B, C and D all lie on the same circle.

20. ★ MOST IMPORTANT THEOREM: CYCLIC QUADRILATERAL

In a cyclic quadrilateral:

The sum of each pair of opposite angles is 180° .

Therefore:

$$\angle A + \angle C = 180^\circ$$

and:

$$\angle B + \angle D = 180^\circ$$



21. EXAMPLE

If a cyclic quadrilateral has:

$$\angle A = 80^\circ$$

then:

$$\angle C = 180^\circ - 80^\circ$$

$$\angle C = 100^\circ$$

Similarly, if:

$$\angle B = 110^\circ$$

then:

$$\angle D = 70^\circ$$

22. CONVERSE OF CYCLIC QUADRILATERAL THEOREM

The converse is also true:

If a pair of opposite angles of a quadrilateral add up to 180° , then the quadrilateral is cyclic.

So:

If

$$\angle A + \angle C = 180^\circ$$

then A, B, C and D are concyclic.

23. IMPORTANT THEOREMS – QUICK LIST

Theorem 1

Equal chords subtend equal angles at the centre.

Theorem 2

Equal central angles subtend equal chords.

Theorem 3

A line from the centre to the midpoint of a chord is perpendicular to the chord.

Theorem 4

A perpendicular from the centre to a chord bisects the chord.

Theorem 5

Equal chords are equidistant from the centre.

Theorem 6

Chords equidistant from the centre are equal.

Theorem 7

The longer chord is closer to the centre.

Theorem 8

Angle subtended by an arc at the centre is twice the angle subtended at a point on the circle.

Theorem 9

A diameter subtends a right angle at any point on the circle.

Theorem 10

Equal angles subtended by the same segment can establish concyclicity.

Theorem 11

Opposite angles of a cyclic quadrilateral are supplementary.

Theorem 12

If opposite angles of a quadrilateral are supplementary, the quadrilateral is cyclic.

These results are consolidated in the chapter summary.



24. CHORDS – QUICK COMPARISON

Property	Result
Equal chords	Equal central angles
Equal central angles	Equal chords
Equal chords	Equal distance from centre
Equal distance from centre	Equal chords
Longer chord	Closer to centre
Centre $\rightarrow$ midpoint of chord	Perpendicular
Centre perpendicular to chord	Bisects chord



25. CENTRAL ANGLE VS ANGLE AT CIRCLE

If the same arc is involved:

At centre:

$$\text{Angle} = \theta$$

At circumference:

$$\text{Angle} = \theta/2$$

Therefore:

Central angle = $2 \times$ angle at circumference

This relationship is extremely important.



26. EXAM-STYLE EXAMPLE

A chord AB subtends an angle of 70° at a point C on the circle.

Find the angle subtended at the centre.

Since:

Central angle = $2 \times$ angle at circumference

$$= 2 \times 70^\circ$$

$$= 140^\circ$$

27. ANOTHER EXAMPLE

AB is a diameter of a circle and C is any point on the circle.

Find $\angle ACB$.

Since a diameter subtends a right angle:

$$\angle ACB = 90^\circ$$

28. CYCLIC QUADRILATERAL EXAMPLE

ABCD is cyclic.

Given:

$$\angle A = 65^\circ$$

Find $\angle C$.

Opposite angles of a cyclic quadrilateral are supplementary.

Therefore:

$$\angle A + \angle C = 180^\circ$$

$$65^\circ + \angle C = 180^\circ$$

$$\angle C = 115^\circ$$

29. HOW TO IDENTIFY A CYCLIC QUADRILATERAL

If you are given a quadrilateral and:

$$\angle A + \angle C = 180^\circ$$

or

$$\angle B + \angle D = 180^\circ$$

you can immediately conclude:

The quadrilateral is cyclic.

30. COMMON MISTAKES

Mistake 1

Thinking every chord passes through the centre.

 **Only a diameter passes through the centre.**

Mistake 2

Confusing radius and diameter.



$$\text{Diameter} = 2 \times \text{Radius}$$

✗ Mistake 3

Forgetting the factor 2.

If the angle at the circumference is 40° :

Central angle = 80°

not 40° .

✗ Mistake 4

Using the cyclic quadrilateral theorem for adjacent angles.

✓ **Opposite angles are supplementary.**

✗ Mistake 5

Thinking the longer chord is farther from the centre.

✓ **The longer chord is closer to the centre.**



31. FORMULA & THEOREM SHEET

● Diameter

$$d = 2r$$

● Central angle and angle at circumference

$$\text{Central angle} = 2 \times \text{angle at circumference}$$

● Angle in a semicircle

$$= 90^\circ$$

● Cyclic quadrilateral

$$\angle A + \angle C = 180^\circ$$

$$\angle B + \angle D = 180^\circ$$

● Chord

Equal chords $\leftrightarrow$ Equal central angles

● Chord distance

Equal chords $\leftrightarrow$ Equal distance from centre

★ 32. MOST IMPORTANT FOR EXAMS

Focus especially on these:

1. ★ Equal chords and equal central angles
2. ★ Centre-to-chord perpendicular theorem
3. ★ Converse of the chord theorem
4. ★ Equal chords and distance from centre
5. ★ Longer chord is closer to centre
6. ★ Angle at centre = twice angle at circumference
7. ★ Angle in a semicircle = 90°
8. ★ Concyclic points
9. ★ Cyclic quadrilateral
10. ★ Opposite angles of cyclic quadrilateral = 180°
11. ★ Converse of cyclic quadrilateral theorem
12. ★ Proof-based questions involving circles



33. ONE-PAGE REVISION



Circle

All points are at the same distance from the centre.



Chord

Line segment joining two points on a circle.



Diameter

Longest chord passing through the centre.



Minor arc

Central angle $< 180^\circ$.



Major arc

Central angle $> 180^\circ$.



Equal chords

→ Equal central angles.



Equal chords

→ Equal distances from centre.



Longer chord

→ Closer to centre.



Centre to midpoint of chord

→ Perpendicular to chord.



Central angle

→ Twice angle at circumference.

★ Diameter

→ Subtends 90° .

★ Cyclic quadrilateral

→ Opposite angles add to 180° .

★ Opposite angles = 180°

→ Quadrilateral is cyclic.



Chapter 5 in one sentence

Chapter 5 uses the geometry of circles, chords, arcs and angles to develop important theorems about equal chords, perpendiculars, angles in circles and cyclic quadrilaterals.