Class 9 Maths

Midpoint Theorem

Chapter 12

Ganita Manjari

Statement

The line segment joining the midpoints of two sides of a triangle is **parallel** to the third side and is equal to **half** of it.

Given

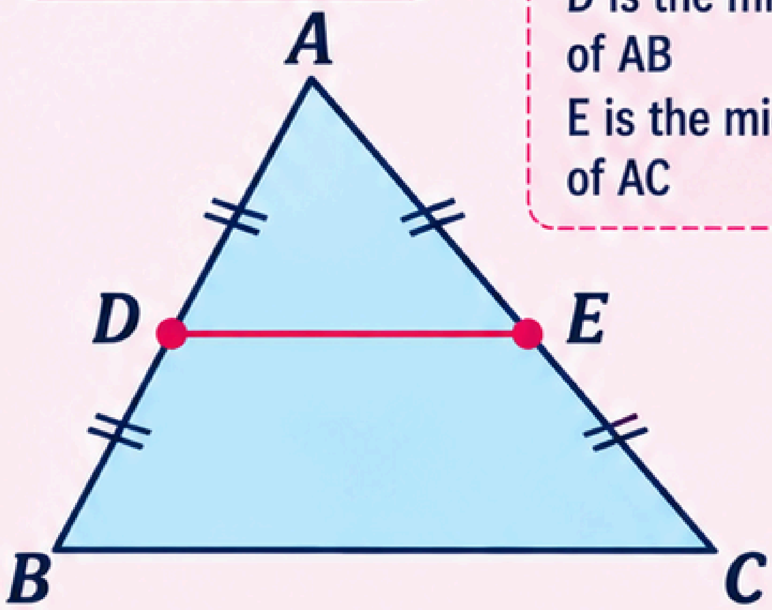
In $\triangle ABC$, D and E are the midpoints of sides AB and AC respectively.

To Prove

(i) $DE \parallel BC$

(ii) $DE = \frac{1}{2} BC$

Diagram



D is the midpoint of AB
E is the midpoint of AC

Proof (Idea)

1. Through point D, draw a line parallel to BC, meeting AC at E.
2. Using parallel lines and the Basic Proportionality Theorem (Thales), we get $AE = EC$.
3. Hence, E is the midpoint of AC and $DE \parallel BC$.
4. Again, using BPT, we get $DE = \frac{1}{2} BC$.

Hence, proved.

Key Points

1. The segment joining the midpoints of two sides of a triangle is parallel to the third side.
2. Its length is equal to half the length of the third side.
3. This theorem is useful in solving many geometry problems, including those on areas and parallel lines.

Small Steps Lead to Big Solutions!

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Converse of Midpoint Theorem

Chapter 12

Statement

If a line segment joining the **midpoints** of two sides of a triangle is **parallel** to the third side, then it is equal to **half of the third side**.

Given

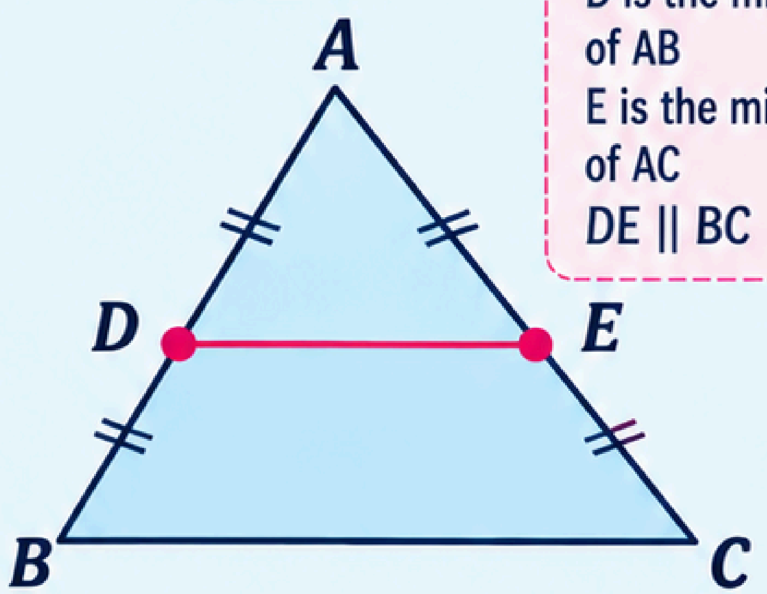
In $\triangle ABC$, D and E are the midpoints of sides AB and AC respectively, and

$$DE \parallel BC$$

To Prove

$$DE = \frac{1}{2} BC$$

Diagram



D is the midpoint of AB
E is the midpoint of AC
DE || BC

Example

In $\triangle ABC$, D and E are the midpoints of AB and AC. If BC = 14 cm and DE || BC, find the length of DE.

Solution:

$$DE = \frac{1}{2} BC$$
$$= \frac{1}{2} \times 14$$
$$= 7 \text{ cm}$$

Therefore, DE = 7 cm.

Key Points

1. D and E are the midpoints of two sides of a triangle.
2. If the segment joining the midpoints is parallel to the third side, then its length is half of the third side.
3. This is the converse of the Midpoint Theorem.
4. Useful in proving that a segment is a mid-segment in a triangle.

Parallel lines help us find equal lengths!

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Centroid Theorem

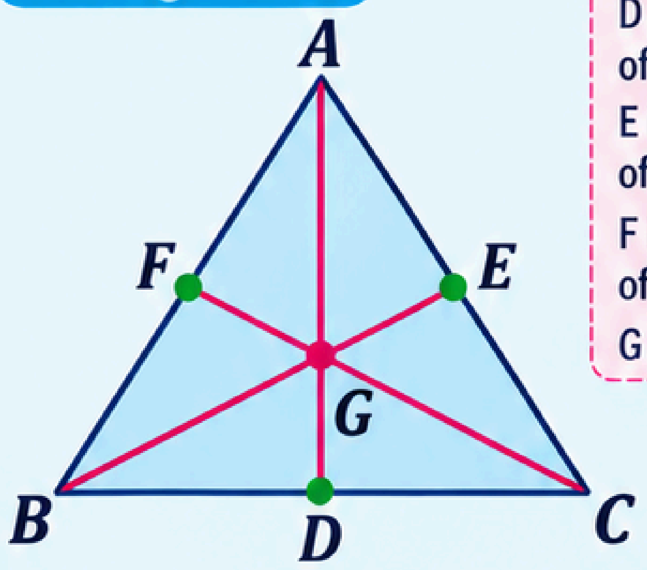
Chapter 12

Ganita Manjari

Statement

The three medians of a triangle are concurrent at a point called the **centroid**, and the centroid divides each median in the **ratio 2 : 1**, measured from the vertex.

Diagram



D is the midpoint of BC
E is the midpoint of AC
F is the midpoint of AB
G is the centroid

In Symbols

In ΔABC, AD, BE and CF are the medians and they are concurrent at the centroid G.

$$AG : GD = BG : GE = CG : GF = 2 : 1$$

That is,

$$AG = 2GD, \quad BG = 2GE, \quad CG = 2GF$$

Example

In ΔABC, AD is a median and G is the centroid. If $AD = 9$ cm, find AG and GD.

Solution:

Since $AG : GD = 2 : 1$

Let $AG = 2x$ and $GD = x$

Then, $AD = AG + GD = 2x + x = 3x$

Given $AD = 9$ cm $\Rightarrow 3x = 9 \Rightarrow x = 3$

So, $AG = 2x = 6$ cm and $GD = x = 3$ cm

Therefore, $AG = 6$ cm and $GD = 3$ cm.

Key Points

1. The medians of a triangle meet at a single point called the **centroid**.
2. The centroid divides each median in the **ratio 2 : 1** (from the vertex).
3. The centroid is represented by G.
4. It is used in many geometrical problems and in finding the position of the centre of a triangle.

Three medians, One centroid, Many possibilities!

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Class 9 Maths

Varignon's Theorem

Chapter 12

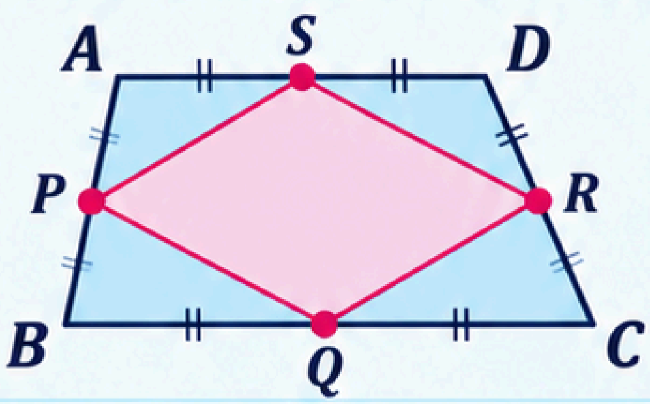
Ganita Manjari

Statement

The line segments joining the midpoints of the sides of a quadrilateral, taken in order, form a **parallelogram**.

Given

In quadrilateral $ABCD$, P, Q, R and S are the midpoints of AB, BC, CD and DA respectively.



P is the midpoint of AB
Q is the midpoint of BC
R is the midpoint of CD
S is the midpoint of DA

To Prove

The quadrilateral $PQRS$ formed by joining the midpoints of the sides of a quadrilateral $ABCD$ is a parallelogram.

i.e., $PQ \parallel SR$ and $PS \parallel QR$

(Opposite sides are parallel and equal)

Example

In quadrilateral $ABCD$, P, Q, R and S are the midpoints of the sides (as shown). Show that $PQRS$ is a parallelogram.

Solution:

1. In $\triangle ABC$, P and Q are midpoints of AB and BC.
   So, $PQ \parallel AC$ and $PQ = \frac{1}{2} AC$. ...(1)
2. In $\triangle ADC$, S and R are midpoints of AD and DC.
   So, $SR \parallel AC$ and $SR = \frac{1}{2} AC$. ...(2)
3. From (1) and (2), we get $PQ \parallel SR$ and $PQ = SR$.
   Similarly, $PS \parallel QR$ and $PS = QR$.

Hence, $PQRS$ is a parallelogram.

Key Points

1. P, Q, R and S are the midpoints of the sides of a quadrilateral.
2. The quadrilateral formed by joining these midpoints in order is a parallelogram.
3. Opposite sides of the midpoint quadrilateral are parallel and equal.
4. This theorem is known as **Varignon's Theorem** (named after the French mathematician Pierre Varignon).

A beautiful result with many applications!

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