Class 9 Maths

The World of Algorithms

Chapter 11

Ganita Manjari

What is an Algorithm?

An **algorithm** is a well-defined, step-by-step procedure to solve a problem or perform a task.

Key Features of an Algorithm

1. **Well-defined steps**
   Each step is clear and unambiguous.
2. **Finite number of steps**
   The algorithm must terminate after a limited number of steps.
3. **Input**
   It takes zero or more inputs.
4. **Output**
   It produces at least one output.
5. **Effective steps**
   Each step is basic and can be carried out in a finite amount of time.

Example of an Algorithm

Algorithm to Find the Sum of Two Numbers

Step 1: Start

Step 2: Take two numbers, a and b as input.

Step 3: Add the numbers: $S = a + b$

Step 4: Display the sum S.

Step 5: Stop.

Real-Life Examples of Algorithms

- A recipe to make a dish
- Steps to find a route on a map
- Using a calculator to add numbers
- Steps to unlock a phone
- Steps to solve a mathematical problem

Why Are Algorithms Important?

- Help us solve problems in a logical way.
- Used in computers and everyday life.
- Save time and reduce errors.
- Can be applied to many different situations.

A small step for a big solution!

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Chapter 11

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Algorithm to Add Two Numbers

An algorithm is a step-by-step procedure. The following algorithm helps us to find the sum of two numbers.

Algorithm (Step-by-Step)

1. **Start** Begin the algorithm.
2. **Input** Take two numbers, a and b, as input.
3. **Process** Add the two numbers. Compute $S = a + b$.
4. **Output** Display the sum S.
5. **Stop** End the algorithm.

Example

Find the sum of 23 and 17 using the algorithm.

Step 1: Start

Step 2: Input: $a = 23, \; b = 17$

Step 3: Process: $S = a + b$

$S = 23 + 17$

$S = 40$

23 + 17 = 40

Step 4: Output: The sum is 40.

Step 5: Stop.

Key Points

- The algorithm has a finite number of steps.
- Each step is clear and unambiguous.
- It takes inputs and produces output.
- The same steps work for any pair of numbers.
- It helps us solve problems in a systematic way.

Try Yourself

Use the above algorithm to find the sum of:

1. 56 and 28
2. 123 and 77
3. 999 and 1
4. Any two numbers of your choice

Same steps Work for all numbers!

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Chapter 11

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Algorithm to Find the Divisors of a Number

A **divisor** of a number is a whole number that divides it exactly, leaving no remainder.

Algorithm (Step-by-Step)

1. **Start**
   Begin the algorithm.
2. **Input**
   Take a positive whole number n as input.
3. **Process**
   Check all whole numbers from 1 to n.
4. If a number divides n exactly (with remainder 0), then it is a divisor. Add it to the list.
5. **Output**
   Display the list of all divisors.
6. **Stop**
   End the algorithm.

Example

Find all the divisors of 12 using the algorithm.

Step 1: Start

Step 2: **Input:** $n = 12$

Step 3: Check numbers from 1 to 12:

$12 \div 1 = 12$ ✓	$12 \div 7 = 1$ remainder 5 ✗
$12 \div 2 = 6$ ✓	$12 \div 8 = 1$ remainder 4 ✗
$12 \div 3 = 4$ ✓	$12 \div 9 = 1$ remainder 3 ✗
$12 \div 4 = 3$ ✓	$12 \div 10 = 1$ remainder 2 ✗
$12 \div 5 = 2$ ✗	$12 \div 11 = 1$ remainder 1 ✗
$12 \div 6 = 2$ ✓	$12 \div 12 = 1$ ✓

Step 4: **Output:** Divisors of 12 are **1, 2, 3, 4, 6, 12.**

Step 5: Stop

Key Points

- ✓ A divisor divides the number exactly (remainder 0).
- ✓ Every number has at least two divisors: 1 and itself.
- ✓ The number of divisors is finite.
- ✓ This algorithm works for any positive whole number.
- ✓ Divisors always come in pairs (except for perfect squares).

Try Yourself

Find the divisors of:

1. 18
2. 25
3. 36
4. 50

Run the steps and find out!

Algorithms make problems easier!

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Chapter 11

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Algorithm to Check a Prime Number

A **prime number** is a natural number greater than 1 that has exactly two distinct factors: 1 and the number itself.

Algorithm (Step-by-Step)

1. **Start**
   Begin the algorithm.
2. **Input**
   Take a natural number n ($n > 1$) as input.
3. **Process**
   Check all numbers from 2 to $n - 1$.
   If any of them divides n exactly, then n is not a prime number.
4. **Output**
   If no number from 2 to $n - 1$ divides n, then n is a prime number.
5. **Stop**
   End the algorithm.

Example

Check whether 17 is a prime number using the algorithm.

Step 1: Start

Step 2: **Input:** $n = 17$

Step 3: Check numbers from 2 to 16:

17 is not divisible by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 or 16.

Step 4: **Output:** 17 is a prime number.

Step 5: Stop

Key Points

- A prime number has exactly two factors: 1 and itself.
- Every number greater than 1 is either prime or composite.
- The algorithm checks divisibility by all numbers from 2 to $n - 1$.
- If any number divides n, then n is not prime.
- If no number divides n, then n is a prime number.
- This algorithm works for any natural number greater than 1.

Try Yourself

Use the algorithm to check whether the following numbers are prime or not:

1. 13
2. 24
3. 29
4. 35
5. 41

Apply the steps and find out!

A clear algorithm for a smarter tomorrow!

Think logically, solve easily!

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Chapter 11

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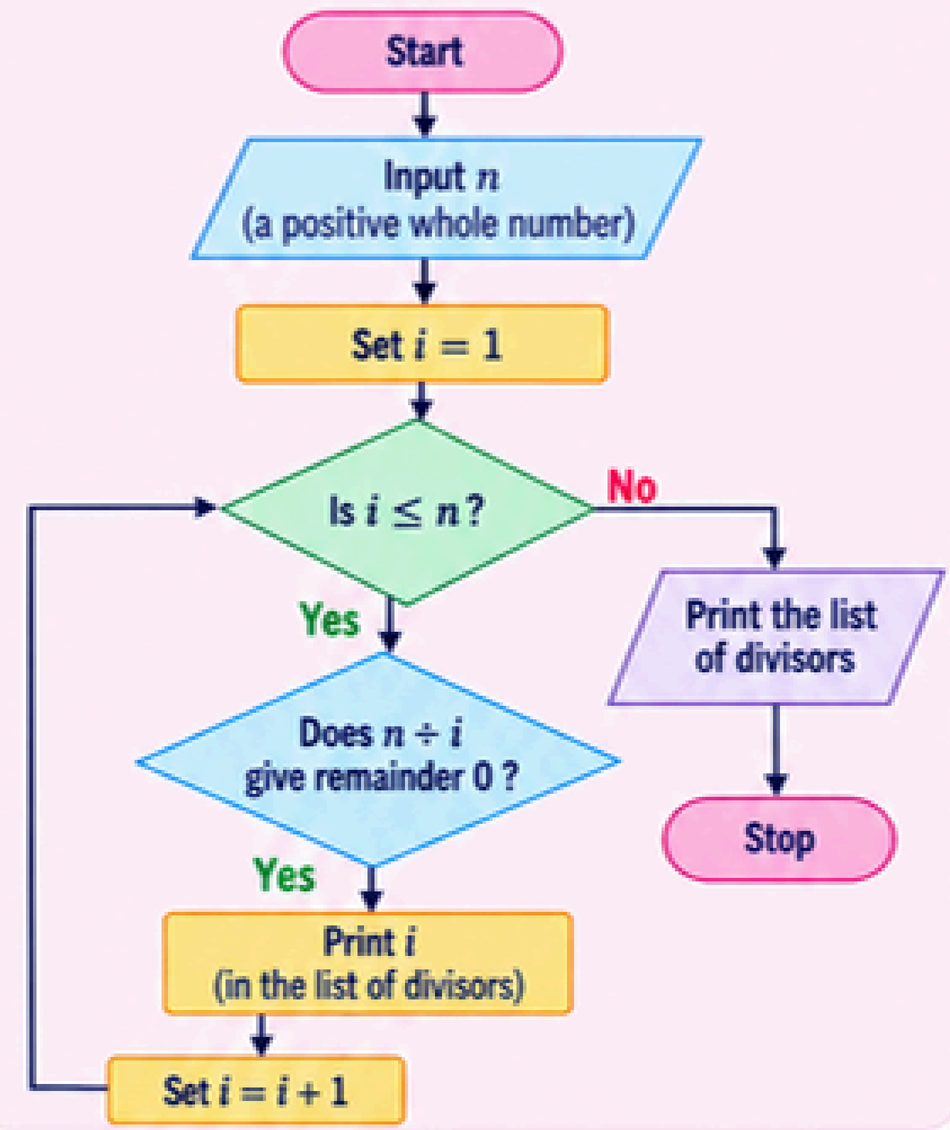
Flowchart to Find the Divisors of a Number

A **flowchart** shows the steps of an algorithm using symbols.
It helps us to find all the divisors of a given number.

Flowchart



Example: Divisors of 12

Let's find all the divisors of 12 using the flowchart.

Step	Value of i	Is $12 \div i$ divisible?	Action
1	1	Yes	Print 1
2	2	Yes	Print 2
3	3	Yes	Print 3
4	4	Yes	Print 4
5	5	No	(no action)
6	6	Yes	Print 6
7	7	No	(no action)
8	8	No	(no action)
9	9	No	(no action)
10	10	No	(no action)
11	11	No	(no action)
12	12	Yes	Print 12

Divisors of 12 are 1, 2, 3, 4, 6, 12.

We only need to check up to n, but we can also stop at $\sqrt{n}$ for a more efficient method.

Same steps work for any positive number!

Key Points

- A flowchart uses standard symbols (Start/Stop, Input/Output, Process, Decision).
- This flowchart finds all divisors of a positive whole number.
- It checks each number from 1 to n.
- A number is a divisor if it leaves remainder 0.
- The list of divisors is displayed at the end.
- The same algorithm works for any positive whole number.

Symbols Used in a Flowchart

Symbol	Name	Meaning
	Terminator (Start/Stop)	Shows the beginning or end
	Input/Output	Used to take input or display output
	Process	Shows an instruction or calculation
	Decision	A condition (Yes/No)
$\rightarrow$ $\downarrow$	Flow line	Shows the direction of the flow

Follow the steps, Find the solution!

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Algorithm to Find the HCF (Highest Common Factor)

The **Highest Common Factor (HCF)** of two or more numbers is the greatest number that divides each of them exactly.

Algorithm (Step-by-Step)

1. **Start**
   Begin the algorithm.
2. **Input**
   Take two positive integers, a and b (as input, with $a \geq b$).
3. **Divide**
   Divide a by b and find the remainder r.
4. **Check**
   If $r = 0$, then b is the HCF. Go to Step 6.
5. **Repeat**
   Otherwise, replace a with b and b with r, and go back to Step 3.
6. **Output**
   Display the HCF.
7. **Stop**
   End the algorithm.

Example

Find the HCF of 252 and 105 using the algorithm.

Step 1:	Start
Step 2:	**Input:** $a = 252, \ b = 105$
Step 3:	**Divide** 252 by 105: $252 = 105 \times 2 + 42$ (remainder 42)
Step 4:	Since remainder $\neq 0$, continue.
Step 5:	Replace $a = 105, \ b = 42$
Step 3:	**Divide** 105 by 42: $105 = 42 \times 2 + 21$ (remainder 21)
Step 5:	Replace $a = 42, \ b = 21$
Step 3:	Divide 42 by 21: $42 = 21 \times 2 + 0$ (remainder 0)
Step 6:	**Output:** HCF = 21
Step 7:	Stop

Repeat until the remainder is 0!

Key Points

- The HCF of two or more numbers is the greatest number that divides each of them exactly.
- The algorithm uses repeated division.
- At each step, replace the larger number by the remainder until the remainder is 0.
- The last non-zero remainder is the HCF.
- This is known as the Euclid's Division Algorithm.

Try Yourself

Find the HCF of the following pairs of numbers using the algorithm.

1. 48 and 18
2. 196 and 84
3. 135 and 45
4. 462 and 107
5. 270 and 192

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Algorithms to Find LCM and HCF

The Least Common Multiple (LCM) and Highest Common Factor (HCF) can be found using step-by-step algorithms.

Algorithm to Find LCM
(Using Prime Factorisation)

1. **Start**
   Begin the algorithm.
2. **Find prime factors**
   Find the prime factors of each number.
3. **Take highest powers**
   For each prime factor, take the highest power that appears in any of the numbers.
4. **Multiply**
   Multiply these highest powers to get the LCM.
5. **Stop**
   End the algorithm.

Example: Find the LCM of 12 and 18

Prime factors:
$12 = 2^2 \times 3$
$18 = 2 \times 3^2$

Take highest powers:
2^2 and 3^2

$\text{LCM} = 2^2 \times 3^2 = 4 \times 9 = \mathbf{36}$

Algorithm to Find HCF
(Using Prime Factorisation)

1. **Start**
   Begin the algorithm.
2. **Find prime factors**
   Find the prime factors of each number.
3. **Take common factors**
   Take only the common prime factors with the smallest powers.
4. **Multiply**
   Multiply these common factors to get the HCF.
5. **Stop**
   End the algorithm.

Example: Find the HCF of 12 and 18

Prime factors:
$12 = 2^2 \times 3$
$18 = 2 \times 3^2$

Common factors with smallest powers:
2^1 and 3^1

$\text{HCF} = 2 \times 3 = \mathbf{6}$

Key Points

- LCM is the smallest common multiple of two or more numbers.
- HCF is the largest common factor of two or more numbers.
- Both algorithms are based on prime factorisation.
- These algorithms are useful in real-life problems.

Try Yourself

Find the LCM and HCF of the following pairs:

1. 15 and 25
2. 24 and 36
3. 30 and 45

Use the steps and find out!

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Algorithm to Find the HCF of Two Numbers

The **Highest Common Factor (HCF)** of two numbers is the largest number that divides both numbers exactly.

Algorithm (Step-by-Step)

1. **Start**
   Begin the algorithm.
2. **Input**
   Take two positive integers, a and b, with $a \geq b$.
3. **Divide**
   Divide a by b and find the remainder r.
4. **Replace**
   If $r \neq 0$, replace a with b and b with r. Go back to Step 3.
5. **Output**
   If $r = 0$, then b is the HCF.
6. **Stop**
   End the algorithm.

Example

Find the HCF of 252 and 105 using the algorithm.

Step 1:	Start
Step 2:	**Input:** $a = 252,\ b = 105$
Step 3:	252 ÷ 105 = 2, remainder 42
Step 4:	**Replace:** $a = 105,\ b = 42$
Step 5:	105 ÷ 42 = 2, remainder 21
Step 6:	**Replace:** $a = 42,\ b = 21$
Step 7:	42 ÷ 21 = 2, remainder 0
Step 8:	**Output:** HCF is 21
Step 9:	Stop

HCF = 21

Key Points

- HCF is the largest number that divides both numbers exactly.
- The algorithm is based on repeated division.
- The remainder becomes the new divisor in each step.
- The process stops when the remainder is 0.
- The last non-zero remainder (or divisor) is the HCF.
- This method works for any two positive integers.

Try Yourself

Find the HCF of the following pairs of numbers using the algorithm.

1. 30 and 18
2. 84 and 36
3. 196 and 56
4. 81 and 27
5. 72 and 48

Follow the steps and find the HCF!

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Euclid's Division Algorithm

For any two positive integers a and b ($b > 0$), there exist unique integers q and r such that $a = bq + r$, where $0 \leq r < b$.

Algorithm (Step-by-Step)

1. **Start**
   Begin the algorithm.
2. **Input**
   Take two positive integers a and b (with $a \geq b$).
3. **Divide**
   Divide a by b to get quotient q and remainder r.
4. **Check**
   If $r = 0$, stop. Then $a = bq$.
5. **Otherwise**
   Replace a with b and b with r. Go back to Step 3.
6. **Output**
   When $r = 0$, the last non-zero remainder is the HCF of the two numbers.
7. **Stop**
   End the algorithm.

Example

Find the HCF of 135 and 45 using Euclid's Division Algorithm.

Step 1:	Start
Step 2:	**Input:** $a = 135$, $b = 45$
Step 3:	**Divide** 135 by 45: $135 = 45 \times 3 + 0$ (remainder 0)
Step 4:	Since remainder is 0, stop.
Step 5:	**Output:** HCF = 45
Step 6:	Stop

When the remainder is 0, the last non-zero remainder is the HCF!

Find the HCF of 196 and 84 using Euclid's Division Algorithm.

Step 1:	Start
Step 2:	**Input:** $a = 196$, $b = 84$
Step 3:	$196 = 84 \times 2 + 28$ ($r = 28$)
Step 4:	$84 = 28 \times 3 + 0$ ($r = 0$)
Step 5:	**Output:** HCF = 28
Step 6:	Stop

Repeat the division until the remainder is 0!

Key Points

- Euclid's Division Algorithm is based on the Division Lemma ($a = bq + r, 0 \leq r < b$).
- It uses repeated division.
- The last non-zero remainder is the HCF.
- It works for any two positive integers.
- It is an efficient method to find the HCF of large numbers.

Try Yourself

Find the HCF of the following pairs using Euclid's Division Algorithm.

1. 72 and 30
2. 150 and 60
3. 252 and 105
4. 308 and 84
5. 315 and 135

Use the algorithm and find the HCF!

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Finding HCF using Euclid's Division Algorithm

Euclid's Division Algorithm is a simple and efficient method to find the **Highest Common Factor (HCF)** of two positive integers.

Steps of the Algorithm

1. **Start**
   Take two positive integers, a and b $(a \geq b)$.
2. **Divide**
   Divide a by b to get quotient q and remainder r.
3. **Check**
   If $r = 0$, then b is the HCF. Stop.
4. **Otherwise**
   Replace a with b and b with r and go back to Step 2.
5. **Repeat**
   Continue this process until the remainder is 0.

Example 1: Find HCF of 252 and 105

Step 1: $252 \div 105 = 2$, remainder 42
$252 = 105 \times 2 + 42$

Step 2: $105 \div 42 = 2$, remainder 21
$105 = 42 \times 2 + 21$

Step 3: $42 \div 21 = 2$, remainder 0
$42 = 21 \times 2 + 0$

Step 4: Since remainder is 0, the HCF is 21.

Repeat until the remainder is 0!

Example 2: Find HCF of 196 and 84

Step 1: $196 \div 84 = 2$, remainder 28
$196 = 84 \times 2 + 28$

Step 2: $84 \div 28 = 3$, remainder 0
$84 = 28 \times 3 + 0$

Step 3: The HCF is 28.

The last non-zero remainder is the HCF!

Key Points

- Based on the Division Lemma: $a = bq + r, 0 \leq r < b$.
- Used to find the HCF of two positive integers.
- The last non-zero remainder is the HCF.
- Works for any two positive integers.
- It is an efficient and systematic method.

Try Yourself

Find the HCF of the following pairs using Euclid's Division Algorithm.

1. 135 and 45
2. 210 and 56
3. 308 and 77
4. 462 and 107
5. 825 and 275

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Algorithm to Find the LCM (Lowest Common Multiple)

The **Lowest Common Multiple (LCM)** of two or more numbers is the smallest positive number that is a multiple of each of them.

Algorithm (Step-by-Step)

1. **Start**
   Begin the algorithm.
2. **Input**
   Take two positive integers, a and b (as input, with $a \geq b$).
3. **Find multiples**
   List the multiples of a and b.
4. **Identify**
   Find the smallest common multiple.
5. **Output**
   Display the LCM.
6. **Stop**
   End the algorithm.

Example

Find the LCM of 12 and 18 using the algorithm.

Step 1:	Start
Step 2:	**Input:** $a = 12$, $b = 18$
Step 3:	**List multiples:** Multiples of 12: 12, 24, 36, 48, 60, 72, ... Multiples of 18: 18, 36, 54, 72, ...
Step 4:	Smallest common multiple = 36
Step 5:	**Output:** LCM = 36
Step 6:	Stop

36 is the smallest common multiple!

Key Points

- The LCM of two or more numbers is the smallest positive number that is a multiple of each of them.
- The algorithm uses the idea of multiples.
- There can be infinitely many common multiples, but the LCM is the smallest one.
- The algorithm works for any two positive integers.
- LCM is useful in real-life problems (e.g., finding common time intervals).

Try Yourself

Find the LCM of the following pairs using the algorithm.

1. 6 and 8
2. 15 and 20
3. 9 and 12
4. 14 and 21
5. 16 and 24

Use the steps and find out!

Think logically
Solve step by step!

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Aryabhata's Division Algorithm

For any two positive integers a and b ($b > 0$), there exist unique integers q and r such that $a = bq + r$, where $0 \leq r < b$.

What is it?

Aryabhata's Division Algorithm states that when a positive integer a is divided by another positive integer b ($b > 0$), we get a unique quotient q and a unique remainder r such that

$$a = bq + r$$

where $0 \leq r < b$.

Ancient Wisdom, Modern Maths!

Example

Divide 135 by 12 using Aryabhata's Division Algorithm.

Quotient (q)

Divisor

$$\begin{array}{r|l} & 11 \\ \hline 12 & 135 \\ & -132 \\ \hline & 3 \end{array}$$

Dividend (a)

Remainder (r)

So, $135 = 12 \times 11 + 3$
Here, $q = 11$, $r = 3$
and $0 \leq 3 < 12$.

Remainder is less than the divisor.

Another Example

Divide 196 by 15.

$$\begin{array}{r|l} & 13 \\ \hline 15 & 196 \\ & -195 \\ \hline & 1 \end{array}$$

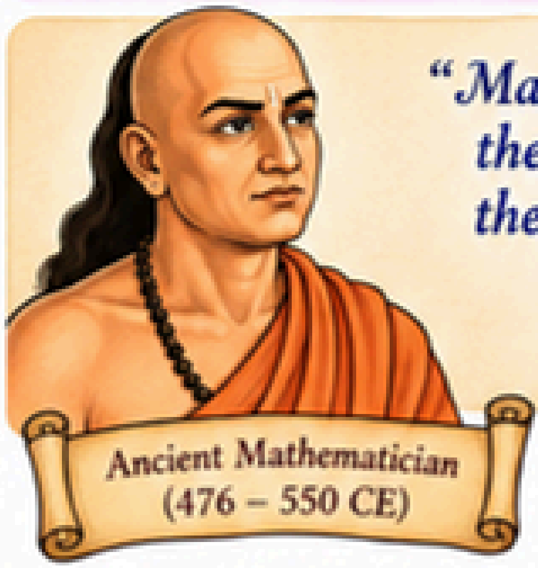
So, $196 = 15 \times 13 + 1$
Here, $q = 13$, $r = 1$
and $0 \leq 1 < 15$.

Steps of the Algorithm

1. Take two positive integers a and b (with $b > 0$).
2. Divide a by b to get quotient q and remainder r.
3. Write $a = bq + r$.
4. Check that $0 \leq r < b$.
5. The quotient q and remainder r are unique.

Key Points

- Works for any two positive integers.
- The quotient and remainder are unique.
- The remainder is always less than the divisor ($0 \leq r < b$).
- Also known as the Division Algorithm.
- Named after the great Indian mathematician Aryabhata.



"Mathematics is the language of the universe."

– Aryabhata

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Euclidean Algorithm vs Aryabhata's Division Algorithm

Both algorithms are used to find the HCF (Highest Common Factor) of two positive integers, but their approaches are slightly different.

Euclidean Algorithm

In Euclid's Algorithm, we repeatedly replace the larger number by the remainder obtained on dividing the larger number by the smaller number, until the remainder becomes 0. The last non-zero remainder is the HCF.

$$\text{HCF}(a, b) = \text{HCF}(b, a \bmod b)$$

Example: Find HCF of 252 and 105

- Step 1: $252 = 105 \times 2 + 42$
- Step 2: $105 = 42 \times 2 + 21$
- Step 3: $42 = 21 \times 2 + 0$
- Step 4: HCF = 21

Key Features

- Generally begins with the larger number.
- Uses repeated division with remainder.
- Very efficient for large numbers.
- Commonly used in modern mathematics and computer algorithms.

Aryabhata's Division Algorithm

In Aryabhata's Algorithm, we write the smaller number as a linear combination of the larger number and a remainder (smaller than the divisor), repeating the process until the remainder becomes 0. The last non-zero remainder is the HCF.

$$\text{Smaller number} = (\text{Larger number} \times q) + r$$

Example: Find HCF of 252 and 105

- Step 1: $105 = 252 \times 0 + 105$
- Step 2: $252 = 105 \times 2 + 42$
- Step 3: $105 = 42 \times 2 + 21$
- Step 4: $42 = 21 \times 2 + 0$

HCF = 21

Both methods give the same HCF!

Key Features

- Can begin with the smaller number.
- Expresses the smaller number in the form $a = bq + r$.
- Based on the Division Algorithm.
- Introduced by the great Indian mathematician Aryabhata (476 – 550 CE).

Comparison at a Glance

Point	Euclidean Algorithm	Aryabhata's Division Algorithm
Starting number	Usually the larger number	Can start with the smaller number
Key idea	Repeated division with remainder	Express smaller number as a linear combination
Formula	$\text{HCF}(a, b) = \text{HCF}(b, a \bmod b)$	$a = bq + r$ $(0 \le r < b)$
Origin	Euclid (Greek mathematician)	Aryabhata (Indian mathematician)
Used for	HCF of two positive integers	HCF of two positive integers
Efficiency	Very efficient, widely used	Equally efficient, especially useful historically

Different paths Same answer!

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GCD Reduction Rule

The GCD Reduction Rule is used to find the HCF (Greatest Common Divisor) of two integers by replacing the larger number with the remainder.

Statement

For any two positive integers a and b (with $a \geq b$),

$$\text{HCF}(a, b) = \text{HCF}(b, r)$$

where r is the remainder when a is divided by b.

Why does it work?

Dividing a by b, we can write

$$a = bq + r \quad (0 \leq r < b)$$

Any common divisor of a and b also divides r. So, a and b have the same common divisors as b and r. Therefore,

$$\text{HCF}(a, b) = \text{HCF}(b, r)$$

Same HCF, smaller numbers!

Steps (GCD Reduction Rule)

1. Divide the larger number by the smaller number.
2. Find the remainder.
3. Replace the larger number with the smaller number and the smaller number with the remainder.
4. Repeat the process until the remainder becomes 0.
5. The last non-zero remainder is the HCF.

Example 1: Find HCF of 252 and 105

Step 1: $252 = 105 \times 2 + 42$

Step 2: $105 = 42 \times 2 + 21$

Step 3: $42 = 21 \times 2 + 0$

$$\text{HCF}(252, 105) = 21$$

We get a sequence of smaller numbers till the remainder is 0.

Example 2: Find HCF of 135 and 48

Step 1: $135 = 48 \times 2 + 39$

Step 2: $48 = 39 \times 1 + 9$

Step 3: $39 = 9 \times 4 + 3$

Step 4: $9 = 3 \times 3 + 0$

$$\text{HCF}(135, 48) = 3$$

The last non-zero remainder is the HCF.

Key Points

- ✓ Based on the Division Algorithm.
- ✓ Repeatedly replaces the larger number with the remainder.
- ✓ Always gives the same HCF.
- ✓ Very useful for large numbers.
- ✓ Also used in the Euclidean Algorithm.

Smaller steps lead to bigger solutions!

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GCD Stopping Condition

In the Euclidean Algorithm (and also in the GCD Reduction Rule), we keep dividing and replacing the larger number with the remainder. We stop when the remainder becomes 0.

What is the Stopping Condition?

The stopping condition is reached when the **remainder becomes 0**.

At this point, the last non-zero remainder is the GCD (HCF) of the two numbers.

Stop when $r = 0$
$\Rightarrow$ Last non-zero remainder is the GCD.

Remainder 0 means we have found the GCD!

Example 1: GCD of 252 and 105

Step 1: $252 = 105 \times 2 + 42$

Step 2: $105 = 42 \times 2 + 21$

Step 3: $42 = 21 \times 2 + 0$

Remainder is 0 → Stop here!

So, GCD(252, 105) = 21
The last non-zero remainder is 21.

Example 2: GCD of 135 and 48

Step 1: $135 = 48 \times 2 + 39$

Step 2: $48 = 39 \times 1 + 9$

Step 3: $39 = 9 \times 4 + 3$

Step 4: $9 = 3 \times 3 + 0$

Remainder is 0 → Stop here!

So, GCD(135, 48) = 3
The last non-zero remainder is 3.

Example 3: GCD of 98 and 56

Step 1: $98 = 56 \times 1 + 42$

Step 2: $56 = 42 \times 1 + 14$

Step 3: $42 = 14 \times 3 + 0$

Remainder is 0 → Stop here!

So, GCD(98, 56) = 14
The last non-zero remainder is 14.

Key Points

- ✓ Keep dividing and replacing the larger number with the remainder.
- ✓ Stop when the remainder becomes 0.
- ✓ The last non-zero remainder is the GCD.
- ✓ This is the stopping condition.
- ✓ Works for any two positive integers.

Remember

- When remainder = 0 → Stop.
- Do not continue further.
- The last non-zero remainder is the GCD.
- This applies to both the Euclidean Algorithm and the GCD Reduction Rule.

Same steps. Same answer. Stronger understanding!

Stop at 0. Find the GCD!

Learn ♥ Practice ♥ Grow

Mathematics grows logical minds!