📘 Class 9 Maths – Ganita Manjari Part II

Chapter 13: Two Variables, One Line – Chapter Notes

Chapter 13 develops **linear equations in two variables**, their solutions and graphs, followed by **pairs of linear equations** and methods for solving them. The chapter also connects algebraic solutions with the geometry of lines.

---

1. Linear Equations in Two Variables

A linear equation in two variables is an equation of the form:

ax + by + c = 0

where:

- a, b and c are real numbers
- a and b are not both zero.

Example

60x + 50y = 280

Here:

- x = kilograms of mangoes
- y = kilograms of bananas

The equation represents the total cost of the two quantities.

Standard form

ax + by + c = 0

A linear equation in two variables has **infinitely many solutions**.

---

2. Solution of a Linear Equation in Two Variables

A **solution** is an ordered pair (x, y) that satisfies the equation.

Example

Consider:

$3x + 2y = 12$

Take:

$x = 2, y = 3$

Then:

$3(2) + 2(3)$

$= 6 + 6$

$= 12$

Therefore,

(2, 3) is a solution.

Similarly,

(4, 0) is also a solution because:

$3(4) + 2(0) = 12$.

But:

(1, 3) is not a solution because:

$3(1) + 2(3)$

$= 3 + 6$

$= 9 \neq 12$.

Important

In an ordered pair:

(x, y)

the first number is the **x-coordinate** and the second number is the **y-coordinate**.

---

3. Finding Solutions

For any chosen value of x, we can calculate the corresponding value of y.

Example

Find two solutions of:

2x + y = 7

Put x = 0:

2(0) + y = 7

y = 7

So,

(0, 7) is a solution.

Put y = 0:

2x + 0 = 7

x = 7/2

So,

(7/2, 0) is another solution.

Since x can take infinitely many values, a linear equation in two variables generally has infinitely many solutions.

---

4. Graph of a Linear Equation

Each solution of a linear equation can be represented as a point on the Cartesian plane.

When all the solutions are plotted, they lie on a **straight line**.

Therefore:

Every point on the graph of a linear equation is a solution of the equation.

And:

Every solution of the equation is represented by a point on its graph.

Example

For:

$3x + 2y = 12$

Rewrite:

$2y = -3x + 12$

Therefore,

$y = -3x/2 + 6$

The graph is a straight line.

---

5. Ordered Pairs and Quadrants

A point is represented as:

(x, y)

The sign of x and y determines the quadrant.

Quadrant	x	y
I	+	+
II	−	+
III	−	−

IV	+	−

A linear equation can have solutions lying in different quadrants.

---

6. Slope of a Line

The **slope** describes the steepness of a line.

It is the ratio:

Slope = Rise / Run

For two points:

(x_1, y_1) and (x_2, y_2)

the slope is:

$m = (y_2 - y_1)/(x_2 - x_1)$

The slope remains the same regardless of which two points on the same line are selected.

Example

For:

A(2, 0) and B(4, 3)

Rise:

3 − 0 = 3

Run:

4 − 2 = 2

Therefore,

m = 3/2.

---

7. Positive, Negative and Zero Slope

Positive slope

If:

m > 0

the line rises from left to right.

Example:

y = 2x + 1

Slope = 2.

For every 1-unit increase in x, y increases by 2.

Negative slope

If:

m < 0

the line falls from left to right.

Example:

y = -3x + 4

Slope = -3.

For every 1-unit increase in x, y decreases by 3.

Zero slope

If:

m = 0

the line is horizontal and parallel to the x-axis.

Example:

y = 5

Slope = 0.

---

8. Slope-Intercept Form

A linear equation can be written as:

y = mx + d

This is called the **slope-intercept form**.

Here:

- **m = slope**
- **d = y-intercept**

The line cuts the y-axis at:

(0, d).

Example

Given:

5x + y = 3

Rewrite:

y = -5x + 3

Therefore:

m = -5

and

d = 3

So the line crosses the y-axis at:

(0, 3).

---

9. Relation Between Standard Form and Slope

Given:

$$ax + by + c = 0$$

Rearrange:

$by = -ax - c$

Therefore,

$$y = -(a/b)x - c/b$$

Comparing with:

$$y = mx + d$$

we get:

$$m = -a/b$$

and

$\mathbf{d = -c/b}$, provided $b \neq 0$.

---

10. Pair of Linear Equations in Two Variables

A pair of linear equations can be written as:

$$a_1x + b_1y + c_1 = 0$$

and

$$a_2x + b_2y + c_2 = 0$$

where:

- a_1, b_1 are not both zero
- a_2, b_2 are not both zero.

A **solution of the pair** is an ordered pair (x, y) that satisfies **both equations simultaneously**.

---

11. Real-Life Situations Leading to a Pair of Equations

Example: Movie tickets and snack boxes

Let:

x = cost of one movie ticket

y = cost of one snack box

If:

2 movie tickets + 3 snack boxes = ₹850

then:

2x + 3y = 850

If:

4 movie tickets + 1 snack box = ₹1100

then:

4x + y = 1100

Thus, we get a pair of linear equations.

---

12. Method of Substitution

The substitution method is used to solve a pair of linear equations.

Steps

1. Select one equation.
2. Express one variable in terms of the other.
3. Substitute this expression into the other equation.
4. Solve for one variable.
5. Substitute its value back to find the other variable.
6. Verify the solution.

Example

Solve:

$7x - 15y = 2$ …(1)

$x + 2y = 3$ …(2)

From (2):

$x = 3 - 2y$

Substitute in (1):

$7(3 - 2y) - 15y = 2$

$21 - 14y - 15y = 2$

$21 - 29y = 2$

$-29y = -19$

$y = 19/29$

Now substitute in:

$x = 3 - 2y$

$x = 3 - 38/29$

$x = 87/29 - 38/29$

$x = 49/29$

Therefore,

$(x, y) = (49/29, 19/29)$.

The textbook then emphasises verifying the obtained solution in both original equations.

---

13. Method of Elimination

In the elimination method, one variable is eliminated by adding or subtracting suitable multiples of the equations.

Basic steps

1. Decide which variable to eliminate.
2. Multiply one or both equations so that the coefficients of that variable become equal or opposites.
3. Add or subtract the equations.
4. Find one variable.
5. Substitute its value into either original equation.
6. Find the other variable.
7. Verify the solution.

Example from the chapter

For a pair of equations representing two persons' incomes and expenditures, the equations are multiplied appropriately so that the y terms become equal:

$27x - 12y = 6000$

$28x - 12y = 8000$

Subtracting:

$x = 2000$

Substituting gives:

$y = 4000$

The corresponding monthly incomes are:

₹18,000 and ₹14,000.

---

14. Graphical Method

A pair of linear equations can also be solved graphically.

Each equation represents a straight line.

The solution of the pair is the point common to both lines.

Three possibilities

1. Intersecting lines

The lines intersect at one point.

Therefore:

One unique solution

2. Parallel lines

The lines never meet.

Therefore:

No solution

3. Coincident lines

The two equations represent the same line.

Therefore:

Infinitely many solutions.

---

15. Example of Graphical Solution

Consider:

$x + 3y = 6$

and

$2x - 3y = 12$

For the first equation:

When $x = 0$:

$3y = 6$

$y = 2$

Point = **(0, 2)**

When $x = 6$:

$y = 0$

Point = **(6, 0)**

For the second equation:

When $x = 0$:

$-3y = 12$

$y = -4$

Point = **(0, −4)**

When $x = 3$:

$6 - 3y = 12$

$y = -2$

Point = **(3, −2)**

The two lines intersect at:

(6, 0)

Therefore,

x = 6, y = 0

is the unique solution.

---

16. Nature of Solutions Using Coefficients

For:

$\mathbf{a_1x + b_1y + c_1 = 0}$

and

$\mathbf{a_2x + b_2y + c_2 = 0}$

compare:

$\mathbf{a_1/a_2}$**,** $\mathbf{b_1/b_2}$ **and** $\mathbf{c_1/c_2}$

Case 1: Unique solution

If:

$\mathbf{a_1/a_2 \neq b_1/b_2}$

the pair has a **unique solution**.

Geometrically, the lines intersect at one point.

Case 2: No solution

If:

$\mathbf{a_1/a_2 = b_1/b_2 \neq c_1/c_2}$

the pair has **no solution**.

Geometrically, the lines are parallel.

Case 3: Infinitely many solutions

If:

$\mathbf{a_1/a_2 = b_1/b_2 = c_1/c_2}$

the pair has **infinitely many solutions**.

Geometrically, the two lines coincide.

17. Algebraic and Geometric Interpretation

Algebraic condition	**Geometric representation**	**Number of solutions**
$a_1/a_2 \neq b_1/b_2$	Intersecting lines	One
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$	Parallel lines	None
$a_1/a_2 = b_1/b_2 = c_1/c_2$	Coincident lines	Infinitely many

18. Important Forms to Remember

Linear equation in two variables

$\mathbf{ax + by + c = 0}$

Slope-intercept form

$\mathbf{y = mx + d}$

Slope

$\mathbf{m = (y_2 - y_1)/(x_2 - x_1)}$

Standard form to slope-intercept form

$\mathbf{y = -(a/b)x - c/b}$

Pair of equations

$\mathbf{a_1x + b_1y + c_1 = 0}$

$\mathbf{a_2x + b_2y + c_2 = 0}$

Nature of solutions

$a_1/a_2 \neq b_1/b_2$ → unique solution

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$ → no solution

$a_1/a_2 = b_1/b_2 = c_1/c_2$ → infinitely many solutions

📝 Chapter Summary

- A linear equation in two variables has the form **$ax + by + c = 0$**.
- It has **infinitely many solutions**.
- Solutions are written as **ordered pairs (x, y)**.
- The graph of a linear equation in two variables is a **straight line**.
- The slope of a line is **rise/run**.
- For two points, **$m = (y_2 - y_1)/(x_2 - x_1)$**.
- The slope-intercept form is **$y = mx + d$**.
- **m** represents slope and **d** represents the y-intercept.
- A pair of linear equations can be solved by:
    - **Substitution**
    - **Elimination**
    - **Graphical method**
- Intersecting lines → **unique solution**.
- Parallel lines → **no solution**.
- Coincident lines → **infinitely many solutions**.
- The coefficient ratios help determine the **nature of the solution**.