📘 Class 9 Maths – Ganita Manjari Part II

Chapter 14: Math of Space: Surface Area and Volume

These notes follow the **Chapter 14 content of the 2026–27 Ganita Manjari Part II textbook**. The chapter covers cuboids, cubes, cylinders, cones, pyramids, spheres, hemispheres, and guesstimate problems.

🔷 1. Introduction

A **solid** is a three-dimensional object that:

- has a **surface**
- occupies **space**
- has measurable dimensions.

In this chapter, we learn to find:

- **Surface Area** – area covered by the surface of a solid.
- **Volume** – amount of three-dimensional space occupied by a solid.

The chapter studies:

1. Cuboids and cubes
2. Right circular cylinders
3. Cones
4. Pyramidal shapes
5. Spheres and hemispheres
6. Areas and volumes around us
7. Guesstimates

🟦 2. Cuboid

A **cuboid** is a three-dimensional version of a rectangle.

It has:

- Length = **l**
- Width = **w**
- Height = **h**

A cuboid has **6 rectangular faces**.

Total Surface Area of a Cuboid

The total surface area is:

TSA = 2(lw + wh + hl)

Volume of a Cuboid

Volume is:

V = l × w × h

or

V = lwh

Volume is measured in **cubic units**, such as:

- cm^3
- m^3
- km^3

🟩 3. Cube

A cube is a special cuboid in which:

length = width = height

Let the side of the cube be **a**.

Total Surface Area

A cube has 6 equal square faces.

TSA = $6a^2$

Volume

V = a^3

Important relationship

For a cube:

Volume = side × side × side

V = a^3

🧊 4. Cube Numbers

Cube numbers are:

1, 8, 27, 64, 125, ...

because:

1^3 = 1

2^3 = 8

3^3 = 27

4^3 = 64

5^3 = 125

Thus, the volume of a cube whose side is **a** is:

a^3

🟨 5. Comparing Surface Area and Volume

Two solids can have the **same volume but different surface areas**.

For example, the textbook compares:

- Cube of side 6 cm
- Cuboid of dimensions 9 cm × 6 cm × 4 cm

Both have volume:

216 cm^3

But their surface areas are different:

Cube:

216 cm^2

Cuboid:

228 cm^2

Therefore, equal volume does **not** necessarily mean equal surface area.

Real-life importance

This idea is useful in:

- packaging
- storage
- heat transfer
- engineering

🟦 6. Right Circular Cylinder

A **right circular cylinder** has:

- two circular bases
- a curved surface
- an axis perpendicular to its circular base.

Let:

- Radius = **r**
- Height = **h**

The chapter distinguishes a **right circular cylinder** from an **oblique cylinder**; the chapter studies only right circular cylinders.

Curved Surface Area of Cylinder

When the curved surface is cut and opened, it forms a rectangle.

Its:

- length = circumference of circular base = **$2\pi r$**
- breadth = **h**

Therefore:

CSA = $2\pi rh$

Cylinder Closed at Both Ends

There are two circular bases.

Area of both bases:

$2\pi r^2$

Therefore:

TSA = $2\pi rh + 2\pi r^2$

or

TSA = $2\pi r(h + r)$

Cylinder Closed at One End

For a container such as a tumbler:

Surface Area = $2\pi rh + \pi r^2$

or

Surface Area = $\pi r(2h + r)$

Volume of Cylinder

$V = \pi r^2 h$

This can also be understood as:

Volume = Area of base × Height

Therefore:

$V = \pi r^2 \times h$

🟠 7. Cone

A **right circular cone** is obtained by rotating a right triangle about one of its perpendicular sides.

The cone has:

- Radius = **r**
- Height = **h**
- Slant height = **l**

These quantities satisfy:

$l^2 = h^2 + r^2$

This follows from the **Baudhayana–Pythagoras theorem**.

🔶 8. Curved Surface Area of a Cone

When the curved surface of a cone is opened, it forms a **sector of a circle**.

The curved surface area is:

CSA = πrl

where:

- r = radius
- l = slant height

🔶 9. Total Surface Area of a Cone

The total surface area includes:

- curved surface
- circular base

Therefore:

TSA = $\pi rl + \pi r^2$

or

TSA = $\pi r(l + r)$

🔶 10. Volume of a Cone

The volume of a cone is:

V = $\frac{1}{3} \pi r^2 h$

or:

V = 1/3 × Area of base × Height

Thus, a cone with the same base radius and height as a cylinder has **one-third the volume of the cylinder**.

The textbook also describes an experiment using salt to verify this relationship.

🟪 11. Pyramidal Shapes

A **pyramidal shape** has:

- a polygonal base
- an apex lying in a different plane
- triangular faces connecting the base to the apex.

The base can be:

- triangle
- square
- or another closed shape.

Volume of a Pyramid

V = 1/3 × Area of base × Height

The same formula applies regardless of the shape of the base.

A cone can be thought of as a pyramid with a **circular base**.

Surface Area of a Pyramid

There is no single formula used for every pyramid.

Find the area of **all the polygonal faces** and add them.

Surface Area = Sum of areas of all faces

⚽ 12. Sphere

A sphere is the three-dimensional equivalent of a circle.

Let its radius be **r**.

Surface Area of a Sphere

$A = 4\pi r^2$

Archimedes showed that the surface area of a sphere is equal to the curved surface area of a cylinder having:

- radius = r
- height = 2r

because:

CSA of cylinder = $2\pi r \times 2r = 4\pi r^2$

🟣 13. Volume of a Sphere

The volume of a sphere is:

$V = 4/3\ \pi r^3$

The textbook connects this formula with the surface area of the sphere:

Volume of sphere = 1/3 × Surface Area of sphere × Radius

Therefore:

$V = 1/3 \times 4\pi r^2 \times r$

$V = 4/3\ \pi r^3$

🟣 14. Hemisphere

A **hemisphere** is formed when a sphere is cut into two equal halves through its centre.

It has:

- one curved surface
- one circular base.

Curved Surface Area of Hemisphere

Since it is half the surface area of a sphere:

CSA = $2\pi r^2$

Total Surface Area of Hemisphere

Total surface area includes:

- curved surface
- circular base

Therefore:

TSA = $2\pi r^2 + \pi r^2$

TSA = $3\pi r^2$

Volume of Hemisphere

A hemisphere is half a sphere.

Therefore:

V = $2/3\ \pi r^3$

📌 15. Formula Chart

Solid	Surface Area	Volume
Cuboid	$2(lw + wh + hl)$	lwh
Cube	$6a^2$	a^3
Cylinder – curved	$2\pi rh$	—
Cylinder – closed both ends	$2\pi r(h + r)$	$\pi r^2 h$

Cylinder – one end closed	$\pi r(2h + r)$	—
Cone – curved	πrl	—
Cone – total	$\pi r(l + r)$	$1/3\ \pi r^2 h$
Pyramid	Sum of areas of polygonal faces	1/3 × Area of base × height
Sphere	$4\pi r^2$	$4/3\ \pi r^3$
Hemisphere – curved	$2\pi r^2$	$2/3\ \pi r^3$
Hemisphere – total	$3\pi r^2$	$2/3\ \pi r^3$

🔢 16. Choosing π

The textbook allows the following approximations in the numerical exercises:

$\pi \approx 22/7$

or

$\pi \approx 3.14$

Use the approximation specified in a question; otherwise, choose an appropriate approximation and state it.

📏 17. Units

Surface Area

Measured in **square units**:

- cm^2
- m^2
- km^2

Volume

Measured in **cubic units**:

- cm^3
- m^3
- km^3

Important conversion used in the chapter:

$1000\ cm^3 = 1$ litre

🌍 18. Areas and Volumes Around Us

The chapter encourages identifying everyday objects resembling:

- cubes
- cuboids
- cylinders
- cones
- spheres
- pyramids.

Examples include:

- gas cylinders
- buckets
- tanks
- cupboards
- containers
- balls
- domes
- pyramidal structures.

The aim is to use mathematical formulas to model real objects.

🧠 19. Guesstimates

A **guesstimate** is a reasonable estimate made when exact information is not available.

A good guesstimate involves:

1. Making an initial guess.
2. Identifying the quantities involved.
3. Making reasonable assumptions.
4. Choosing an appropriate mathematical model.
5. Using approximations.
6. Calculating the estimate.
7. Checking whether the answer is reasonable.

The textbook stresses that **different people may obtain different estimates**, but the reasoning and assumptions used are important.

🍘 20. Example of a Guesstimate

The textbook considers estimating how many golgappas can be served from a cylindrical vessel.

The process involves:

Volume of pani in vessel ÷ estimated volume of pani in one golgappa

But the answer depends on assumptions such as:

- estimated radius of a golgappa
- percentage of the golgappa filled with pani
- approximation of its shape.

Therefore, different reasonable assumptions can produce different estimates.

🎾 21. Packing and Empty Spaces

When estimating how many spheres can fit inside a cuboidal space, simply dividing:

Volume of cuboid ÷ Volume of sphere

does not give the exact number.

Why?

Because spheres leave **gaps** between them.

Therefore, this calculation gives an **upper-bound estimate** rather than the actual number that can necessarily fit.

The textbook discusses different arrangements for packing spheres and notes that different configurations can have different packing efficiencies.

⭐ Quick Revision

Cuboid

TSA = 2(lw + wh + hl)
V = lwh

Cube

TSA = $6a^2$
V = a^3

Cylinder

CSA = 2πrh
TSA = 2πr(h + r)
V = $\pi r^2 h$

Cone

$l^2 = h^2 + r^2$
CSA = πrl
TSA = πr(l + r)
V = 1/3 $\pi r^2 h$

Pyramid

V = 1/3 × Area of base × height

Sphere

Surface Area = $4\pi r^2$
Volume = 4/3 πr^3

Hemisphere

CSA = $2\pi r^2$
TSA = $3\pi r^2$
Volume = $2/3\ \pi r^3$

📚 Chapter Summary

Chapter 14 develops the surface areas and volumes of fundamental 3D shapes:

- **Cuboid**
- **Cube**
- **Right circular cylinder**
- **Right circular cone**
- **Pyramid**
- **Sphere**
- **Hemisphere**

It also connects these formulas with **nets, cross-sections, geometric reasoning, real-life objects, packing, modelling and guesstimation**.