📘 Class 9 Maths – Ganita Manjari

📊 Chapter 10: How Quantities Combine: Understanding Data

1. 📊 Combining Quantities

In earlier classes, we learned that an **average** represents the centre of a collection of data.

In this chapter, we extend the idea of average to situations where different parts of the data may have different sizes or levels of importance.

This leads to the concept of the **weighted average** or **weighted mean**.

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2. ⚖️ Average of Averages

Suppose data is divided into two or more groups.

We **cannot always find the overall average by simply averaging the averages of the groups**.

Example

There are **8 seniors** and **3 juniors** in a badminton academy.

- Average height of seniors = **165.5 cm**
- Average height of juniors = **149.33 cm**

Simply calculating:

(165.5 + 149.33) ÷ 2

does not give the correct average height of all 11 trainees.

The reason is that the two groups do not contain the same number of people.

The correct approach is to first find the total contribution from each group:

Total of seniors = 165.5 × 8

Total of juniors = 149.33 × 3

Then:

Overall average = (Total of seniors + Total of juniors) ÷ Total number of trainees

So,

Overall average = (165.5 × 8 + 149.33 × 3) ÷ (8 + 3)

This gives approximately **161.09 cm**.

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3. 🧮 General Formula for Combining Averages

Suppose:

- Collection 1 has **n** values with average **a**
- Collection 2 has **m** values with average **b**

Then:

Sum of Collection 1 = an

Sum of Collection 2 = bm

Therefore,

Combined average = (an + bm) ÷ (n + m)

This is a **weighted average**, because the group sizes act as the weights.

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4. ⚖️ Weighted Average / Weighted Mean

The weighted mean of values

$\mathbf{x_1, x_2, x_3, \ldots, x_n}$

with corresponding weights

$\mathbf{w_1, w_2, w_3, \ldots, w_n}$

is:

Weighted Mean $\mathbf{= (w_1x_1 + w_2x_2 + w_3x_3 + \ldots + w_nx_n) \div (w_1 + w_2 + w_3 + \ldots + w_n)}$

A weighted average is useful when different parts of the data have different **numbers of values or levels of importance**.

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5. 🥤 Weighted Averages in Mixtures

Weighted averages can be used when combining quantities with different concentrations or values.

The contribution of each part depends on **how much of that part is present**.

Therefore, when dealing with mixtures, we should consider both:

- the value or concentration of each part
- the amount of each part

The weighted mean helps determine the combined result.

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6. ⭐ Weighted Ratings

Weighted averages are also useful in rating systems.

For example, a restaurant may give different importance to:

- 🍽️ Food
- 🛎️ Service
- 🌿 Ambience

If the weights are **4 : 3 : 2**, the combined rating is calculated by multiplying each rating by its corresponding weight and dividing by the sum of the weights.

Combined Rating = (4 × Food rating + 3 × Service rating + 2 × Ambience rating) ÷ 9

The textbook gives an example where the ratings are **5, 3 and 4**, producing a combined rating of approximately **4.11**.

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7. 📚 Weighted Averages in Mark Schemes

Weighted averages can also be used when different assessments have different importance.

For example, if one assessment contributes more towards the final result than another, the assessments can be assigned different weights.

The final score is then calculated using the weighted mean rather than treating every component equally.

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8. 📈 Visualising Data

The chapter then explores different ways of representing data graphically.

The choice of graph depends on **what we want to compare or understand**.

For example, the same expenditure data can be grouped in different ways depending on whether we want to compare:

- different families within each category, or
- different categories within each family.

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9. 📊 Clustered Column/Bar Charts

Clustered charts can be used to compare different groups of data.

The arrangement of the clusters should match the question being investigated.

For example:

Choice 1: Compare different families' expenditure in each category.

Choice 2: Compare category-wise expenditure within each family.

Different arrangements make different comparisons easier.

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10. 🥧 Pie Charts

Pie charts represent quantities as parts of a whole.

Each sector represents a proportion of the total.

However, comparing sectors across different pie charts can sometimes be difficult, especially when their orientations differ.

The chapter introduces another visualisation that can make some of these comparisons easier.

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11. 📊 Stacked Bar Charts

A **stacked bar chart** divides a bar into smaller sections representing different categories.

It can show:

- the individual quantities of different categories
- the total quantity for each group

This makes stacked bars useful when the **total as well as the individual components** are important.

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12. 📊 100% Stacked Bar Charts

In a **100% stacked bar chart**, every complete bar represents **100% of the total** for that group.

The individual sections show the **percentage or proportion** contributed by each category.

For example, if a garden's flowers bloomed:

- **50% in summer**
- **30% in monsoon**
- **20% in winter**

the complete bar represents 100% of that garden's flowers.

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13. 🔍 Comparing Totals and Proportions

A 100% stacked bar chart is useful for comparing **proportions**, but it does not show the actual totals.

Therefore, if two gardens have the same percentage of flowers blooming in summer, we cannot conclude that they had the same number of summer flowers unless their total numbers are also known.

A percentage can be compared within each group, but the actual quantities may be different.

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14. 📊 Converting a Stacked Bar Chart to a 100% Stacked Bar Chart

A stacked bar chart can be converted into a 100% stacked bar chart by calculating the percentage contribution of each component to the total.

For example, the textbook gives electricity consumption data for **2005**:

- Lighting = 360 units
- Cooling = 300 units
- Kitchen appliances = 120 units
- Other appliances = 220 units

Total:

360 + 300 + 120 + 220 = 1000 units

Therefore:

Lighting = 36%

Cooling = 30%

Kitchen appliances = 12%

Other appliances = 22%

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15. 🧠 What Can We Infer from a Graph?

A graph does not always provide enough information to answer every question.

We must carefully examine what the graph represents before making an inference.

For a 100% stacked bar chart, we can usually compare **percentages within each group**, but we may not be able to determine the actual quantities across groups when their totals are unknown.

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16. 📋 Grouping Data

Data can sometimes be grouped according to categories such as **age groups**.

The textbook uses a 100% stacked chart to study how different age groups spend their time on activities such as:

- 😴 Sleep
- 💼 Paid work

- 🏠 Unpaid work and care
- 📚 Learning
- 🧑 Personal care
- 🎮 Leisure, social and travel

Grouping data in this way can reveal patterns that may be hidden when only an overall average is considered.

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17. 💡 Understanding What Data Visualisations Show

Different visualisations highlight different aspects of data.

A good data interpretation requires asking:

- What does the graph show?
- What can be compared?
- What information is hidden?
- Can the actual quantities be determined?
- Are we comparing totals or proportions?

The chapter emphasises that **the choice of visualisation affects the conclusions we can draw from data**.

📝 Quick Revision

⚖️ Weighted Mean

Weighted Mean = Σ(wx) ÷ Σw

where:

- **x** = value
- **w** = corresponding weight

📊 Remember

- Simple average treats values equally.
- Weighted average gives different values different weights.
- **Stacked bar charts** can show components and totals.
- **100% stacked bar charts** compare proportions.
- A 100% stacked bar does **not necessarily show actual quantities**.

- The right visualisation depends on what we want to compare.

📌 Chapter Summary

Chapter 10 develops the concept of **weighted averages** and applies it to combining data, mixtures, ratings and mark schemes. It then introduces **stacked and 100% stacked bar charts** and develops the skill of interpreting what different data visualisations allow us to see—and what they may hide.