



# Class 9 Maths – Ganita Manjari



## Chapter 12: Quadrilaterals

These notes follow the **Ganita Manjari Part II** chapter and its terminology. Chapter 12 begins with the precise meaning of a quadrilateral, then studies parallelograms, midpoint theorems, medians and the centroid, Varignon's theorem, and tiling the plane with quadrilaterals.

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### 1. What Exactly Is a Quadrilateral?

A quadrilateral is more than simply a figure with four straight sides. The chapter first examines different possible figures and asks which ones should actually be called quadrilaterals.

The chapter considers issues such as:

- **Convex quadrilaterals**
- **Non-convex quadrilaterals**
- **Self-intersecting quadrilaterals**
- **Planar and non-planar figures**

This leads to the need for a precise definition rather than relying only on the informal idea of "a figure with four sides."

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### 2. Types of Quadrilaterals

#### **Convex Quadrilateral**

A quadrilateral in which the figure does not bend inward.

#### **Non-Convex Quadrilateral**

A quadrilateral having an inward "dent".

#### **Self-Intersecting Quadrilateral**

A quadrilateral in which two of its sides cross each other.

### **Planar Quadrilateral**

A quadrilateral whose four vertices lie in one plane.

The chapter uses these different cases to explore why the definition of a quadrilateral needs care.

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## **3. Important Parts of a Quadrilateral**

For quadrilateral **ABCD**:

### **Sides**

- AB
- BC
- CD
- DA

### **Opposite Sides**

- AB and CD
- BC and DA

### **Adjacent Sides**

Two sides sharing a common vertex.

### **Diagonals**

- AC
- BD

### **Opposite Angles**

- $\angle A$  and  $\angle C$
  - $\angle B$  and  $\angle D$
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## 4. Sum of Angles of a Quadrilateral

The interior angles of a quadrilateral add up to:

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

This fact is used later to reason about parallelograms and other quadrilaterals.

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## 5. Parallelograms

A **parallelogram** is a quadrilateral whose **opposite sides are parallel**.

Parallelograms include:

- Rhombuses
- Rectangles
- Squares

The chapter asks whether we always need to check parallelism directly to identify a parallelogram, or whether there are other tests.

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## 6. Properties of a Parallelogram

### Theorem 1

If ABCD is a parallelogram:

**(a) Opposite sides are equal**

$$AB = CD$$

and

$$AD = BC$$

**(b) Opposite angles are equal**

$$\angle A = \angle C$$

and

$$\angle B = \angle D$$

**(c) Diagonals bisect each other**

If diagonals AC and BD intersect at E, then:

$$AE = EC$$

and

$$BE = ED$$

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## 7. Converse of the Parallelogram Properties

The chapter examines the converses of the three properties above.

All three converse statements are true.

### **Theorem 2 – Converse of Opposite Sides Property**

If the opposite sides of a quadrilateral are equal, then it is a **parallelogram**.

For example, if:

$$AB = CD$$

and

$$AD = BC$$

then ABCD is a parallelogram.

The proof uses **SSS congruence**.

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## 8. Theorem 3 – Converse of Opposite Angles Property

If the opposite angles of a quadrilateral are equal, then it is a **parallelogram**.

That is, if:

$$\angle A = \angle C$$

and

$$\angle B = \angle D$$

then ABCD is a parallelogram.

The proof uses the fact that:

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

to establish that adjacent angles are supplementary and hence the opposite sides are parallel.

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## 9. Theorem 4 – Converse of the Diagonal Property

### Theorem 4

**A quadrilateral whose diagonals bisect each other is a parallelogram.**

If diagonals AC and BD intersect at E and:

$$AE = EC$$

and

$$BE = ED$$

then ABCD is a parallelogram.

The proof uses **SAS congruence** to establish the required parallel sides.

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## 10. Theorem 5 – Equal Parallel Sides Test

### Theorem 5

A quadrilateral with **one pair of equal and parallel opposite sides** is a parallelogram.

For example, if:

$$AB \parallel CD$$

and

$$AB = CD$$

then ABCD is a parallelogram.

This provides another useful test for identifying a parallelogram.

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## 11. Midpoint Theorem

The chapter then connects quadrilaterals with an important theorem about triangles.

### Theorem 6 – Midpoint Theorem

The line segment joining the **midpoints of two sides of a triangle**:

- is parallel to the third side, and
- has half the length of the third side.

If M and N are the midpoints of two sides of  $\triangle ABC$ , then:

$$MN \parallel BC$$

and

$$MN = \frac{1}{2} BC$$

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## 12. Converse of the Midpoint Theorem

### Theorem 7

The line drawn through the **midpoint of one side of a triangle** and **parallel to another side** bisects the third side.

The resulting segment also has **half the length of the parallel side**.

**In simple words:**

If P is the midpoint of AB and a line through P is drawn parallel to BC, meeting AC at Q, then:

$$AQ = QC$$

and

$$PQ = \frac{1}{2} BC$$

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## 13. Medians of a Triangle

A **median** of a triangle is a line segment joining a vertex to the midpoint of the opposite side.

A triangle has **three medians**.

The chapter investigates whether these three medians meet at a common point.

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## 14. Theorem 8 – Centroid Theorem

### Centroid Theorem

The three medians of a triangle pass through a **common point**.

This point is called the **centroid**.

The centroid divides each median in the ratio:

**2 : 1**

The **longer part** is the part connecting the vertex to the centroid.

For example, if G is the centroid on median AD:

**AG : GD = 2 : 1**

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## 15. Midpoints of a Quadrilateral

Now consider quadrilateral **ABCD**.

Let:

- P = midpoint of AB
- Q = midpoint of BC
- R = midpoint of CD
- S = midpoint of DA

Join:

**PQ, QR, RS and SP**

The resulting quadrilateral **PQRS** has a special property.

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## 16. Theorem 9 – Midpoint Theorem for Quadrilaterals

### Theorem 9

The midpoints of the four sides of a quadrilateral are the vertices of a **parallelogram**.

This parallelogram is called the:

**Varignon Parallelogram**

of the original quadrilateral.

### Why?

Using the Midpoint Theorem in the four triangles formed by the diagonals:

**$PQ \parallel AC$**

and

**$RS \parallel AC$**

Therefore:

**$PQ \parallel RS$**

Similarly:

**$QR \parallel PS$**

Therefore PQRS is a parallelogram.

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## 17. Varignon Parallelogram

The parallelogram formed by joining the midpoints of the four sides of a quadrilateral is called its **Varignon parallelogram**.

It has several interesting properties.

For example, the diagonals of the Varignon parallelogram are related to the diagonals of the original quadrilateral.

The chapter uses this construction later when studying **tiling**.

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## 18. Midpoint Theorem for a Trapezium

The chapter also considers a quadrilateral with one pair of parallel sides, i.e. a **trapezium**.

If:

**$AB \parallel DC$**

and E is the midpoint of AD, then a line through E parallel to AB meets BC at its midpoint F.

Thus:

**$EF \parallel AB$**

and the construction gives a midpoint relationship on the other non-parallel side.

The converse is also considered: if F is the midpoint of BC, then  **$EF \parallel AB$** .

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## 19. Trisecting a Segment

The chapter explores how properties of parallelograms, midpoints and centroids can be used to divide a segment into three equal parts.

For example, if ABCD is a parallelogram and M and N are the midpoints of AB and CD, then segments **DM** and **BN** trisect diagonal AC.

The chapter then asks how these ideas can be used to trisect any given segment.

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## 20. Tiling the Plane

One of the major applications in the chapter is **tiling**.

A plane is tiled when copies of a shape can be arranged so that:

- There are no gaps.
- The copies do not overlap.
- The arrangement can continue across the plane.

The chapter investigates whether a given quadrilateral can tile the plane.

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## 21. Tiling with a Parallelogram

A parallelogram can tile the plane.

The chapter explains this using a grid of parallel lines.

Even if the angle between the two sets of parallel lines is changed, the lines remain parallel and the plane continues to be tiled by copies of the parallelogram.

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## 22. Can Any Quadrilateral Tile the Plane?

A key result of the chapter is:

**The plane can be tiled with any given quadrilateral.**

The Varignon parallelogram plays an important role in understanding this result.

The chapter discusses more than one method of constructing such tilings.

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## 23. Tiling Using the Varignon Parallelogram

For a given quadrilateral, first construct its **Varignon parallelogram**.

The Varignon parallelograms can form a grid.

Copies of the original quadrilateral can then be positioned within this structure to construct a tiling.

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## 24. Aperiodic Tilings

The chapter briefly introduces **aperiodic tilings**.

These are tilings that do not repeat in a simple periodic pattern.

The chapter connects aperiodic tilings with the study of **quasicrystals** and mentions the work of **Dan Shechtman**, who received the Chemistry Nobel Prize in 2011 for the discovery of quasicrystals.

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## 25. Important Geometric Reasoning

Chapter 12 emphasises **reasoning**, not just memorising properties.

Important strategies include:

- Using **congruence** to prove geometric facts.
- Working with **converses** of theorems.
- Using the **Midpoint Theorem**.
- Constructing auxiliary lines.
- Using properties of **parallel lines**.
- Using **SAS, SSS and ASA/AAS** reasoning where appropriate.
- Working backwards from a known proof to investigate a converse.

The chapter specifically notes that simply reversing a proof is not always valid; every step in the reverse direction must be justified.

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## 26. Important Theorems – Quick List

### Theorem 1

Properties of a parallelogram:

- Opposite sides are equal.
- Opposite angles are equal.
- Diagonals bisect each other.

### Theorem 2

Equal opposite sides  $\rightarrow$  parallelogram.

### Theorem 3

Equal opposite angles  $\rightarrow$  parallelogram.

### Theorem 4

Diagonals bisect each other  $\rightarrow$  parallelogram.

### Theorem 5

One pair of equal and parallel opposite sides  $\rightarrow$  parallelogram.

### Theorem 6

Midpoint Theorem.

### Theorem 7

Converse of the Midpoint Theorem.

### Theorem 8

Centroid Theorem: medians meet at a point dividing each median in **2 : 1**.

### Theorem 9

Midpoints of the four sides of a quadrilateral form a **parallelogram (Varignon parallelogram)**.

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## 27. Important Terms

**Quadrilateral** – a four-sided figure requiring a precise geometric definition.

**Convex quadrilateral** – a quadrilateral without an inward indentation.

**Non-convex quadrilateral** – a quadrilateral with an inward indentation.

**Self-intersecting quadrilateral** – a quadrilateral whose sides intersect.

**Parallelogram** – a quadrilateral whose opposite sides are parallel.

**Diagonal** – a segment joining opposite vertices.

**Midpoint** – the point dividing a segment into two equal parts.

**Median** – a segment joining a vertex of a triangle to the midpoint of the opposite side.

**Centroid** – the common point of the three medians of a triangle.

**Varignon parallelogram** – the parallelogram formed by joining the midpoints of the four sides of a quadrilateral.

**Tiling** – covering the plane with copies of a shape without gaps or overlaps.

**Aperiodic tiling** – a tiling without a simple repeating periodic pattern.

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## 28. Quick Revision

- The angles of a quadrilateral add up to  **$360^\circ$** .
  - A parallelogram has opposite sides parallel.
  - Opposite sides of a parallelogram are equal.
  - Opposite angles are equal.
  - Diagonals bisect each other.
  - Each of these three properties has a true converse.
  - One pair of equal and parallel opposite sides is enough to establish a parallelogram.
  - The segment joining two side midpoints of a triangle is parallel to the third side and half its length.
  - The converse of the Midpoint Theorem is also true.
  - The three medians of a triangle meet at the **centroid**.
  - The centroid divides every median in the ratio **2 : 1**.
  - Joining the four side midpoints of any quadrilateral produces a **Varignon parallelogram**.
  - The plane can be tiled using **any given quadrilateral**.
  - The chapter also explores **aperiodic tilings** and their connection with quasicrystals.
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## 29. Chapter Summary

**Chapter 12: Quadrilaterals** develops a deeper understanding of quadrilaterals through precise definitions, parallelogram properties and their converses, midpoint theorems, medians and the centroid.

A central result is **Varignon's theorem**, which states that the midpoints of the four sides of any quadrilateral form a parallelogram. The chapter then uses these ideas to explore **tiling the plane with quadrilaterals**, including a discussion of aperiodic tilings and quasicrystals.