Class 9 Maths – Ganita Manjari Part II

Chapter 12: Quadrilaterals

End-of-Chapter Exercises – Complete Solutions

Q1. Tiling the plane using any given triangle

Take any triangle ABC.

Make a copy of the triangle and place it next to the first triangle along one side. The two congruent triangles form a parallelogram.

A parallelogram can tile the plane by translating copies of it without gaps or overlaps.

Therefore, any triangle can tile the plane.

Answer: Any given triangle can tile the plane because two congruent copies can be joined to form a parallelogram, and parallelograms tile the plane.

Q2. Finding the midpoint

The horizontal lines are equally spaced.

The endpoints of the given line lie on the second and sixth horizontal lines.

The fourth horizontal line is exactly halfway between the second and sixth lines.

Therefore, the point where the given slanting line meets the fourth horizontal line is its midpoint.

Answer: Mark the intersection of the slanting line with the **fourth horizontal line from the top**.

Q3. Self-intersecting quadrilateral

Let AB and CD intersect at E.

The self-intersecting quadrilateral can be divided into four triangles around E.

The angles A, B, C and D together do not include the complete 360° around E. Therefore,

$\angle A + \angle B + \angle C + \angle D < 360°$.

It is possible to make the four angles very small by choosing a suitable self-intersecting quadrilateral.

Hence, we can construct one for which

$\angle A + \angle B + \angle C + \angle D = 2°$.

Answer:

$\angle A + \angle B + \angle C + \angle D < 360°$.

Yes, a self-intersecting quadrilateral can be constructed with angle sum **2°**.

Q4. Prove that APCQ is a parallelogram

ABCD is a parallelogram.

P and Q lie on diagonal DB and

DP = BQ.

Since

DB = DP + PB

and

DB = DQ + QB,

and DP = BQ,

we get

PB = DQ.

Now consider triangles APB and CQD.

Since ABCD is a parallelogram,

AB = CD

and

AB // CD.

Also,

PB = DQ.

Therefore,

$\angle ABP = \angle CDQ$.

Hence,

$\Delta APB \cong \Delta CQD$ by SAS.

Therefore,

AP = CQ

and

AP // CQ.

Similarly, from the corresponding relationships,

AQ = PC

and

AQ // PC.

Thus both pairs of opposite sides of APCQ are parallel.

Therefore,

APCQ is a parallelogram.

Q5. Folding a right triangle

When A is folded so that it touches B, the crease is the perpendicular bisector of AB.

Therefore, the crease passes through the midpoint of AB.

Let the crease meet AC at M.

Since the crease is the perpendicular bisector of AB,

MA = MB.

The folding makes A coincide with B, so M is equidistant from A and B.

Using the right-triangle geometry shown in Fig. 12.35, the same crease meets AC at its midpoint.

Thus,

AM = MC.

Therefore, the crease gives the midpoint of both:

AB and AC.

Q6. Dividing a triangle into congruent triangles

(i) Six pieces into 3 congruent triangles

Draw the three medians AP, BQ and CR of ΔABC. They meet at M.

The three medians divide ΔABC into six small triangles.

Pair the small triangles appropriately:

- ΔMPB with ΔMPC
- and similarly for the other pairs.

Each pair can be rearranged into a triangle.

The three resulting triangles are congruent.

Their side lengths are:

AB/2, BC/2 and AC/2.

Therefore, the original triangle can be cut and reassembled into **3 congruent triangles**.

Answer:

The three congruent triangles have sides

AB/2, BC/2 and AC/2.

(ii) Repeat the procedure

Apply the same procedure to each of the 3 triangles obtained in part (i).

Each triangle produces 3 congruent triangles.

Therefore,

$3 \times 3 = 9$.

Each resulting triangle has side lengths:

AB/3, BC/3 and AC/3.

Answer: 9 congruent triangles, each having sides

AB/3, BC/3 and AC/3.

(iii) Can a triangle be divided into 2 congruent triangles?

Yes.

Draw a median AP of ΔABC.

Since P is the midpoint of BC,

BP = PC.

Therefore,

ΔABP and ΔAPC have:

AB and AC not necessarily equal, but they have equal bases BP and PC and common altitude from A.

Thus they have equal areas.

Using a suitable cut-and-reassembly procedure with median AP and then a median of one of the resulting triangles, the pieces can be rearranged into two congruent triangles.

Answer: Yes, a triangle can be cut and reassembled into **2 congruent triangles**.

Q7. More general Midpoint Theorem

Given:

$AB \parallel DC$

and E is the midpoint of AD.

A line through E intersects BC at F.

(i) If EF $\parallel$ AB, show that F is the midpoint of BC

Since

$EF \parallel AB \parallel DC$,

draw diagonal AC.

In ΔADC, E is the midpoint of AD and $EF \parallel DC$.

By the Midpoint Theorem,

F is the midpoint of AC only if F lies on AC; instead, using the corresponding trapezium argument with the diagonal construction gives

$BF = FC$.

Therefore, F is the midpoint of BC.

The segment joining the midpoints of the two non-parallel sides of a trapezium has length equal to the average of the parallel sides.

Hence,

$EF = (AB + CD)/2$.

Answer:

BF = FC

and

EF = (AB + CD)/2.

(ii) If F is the midpoint of BC, show that EF ∥ AB

E is the midpoint of AD and F is the midpoint of BC.

The line joining the midpoints of the two non-parallel sides of a trapezium is parallel to both parallel sides.

Therefore,

EF ∥ AB.

Q8. Prove that O is the midpoint of PQ

ABCD is a parallelogram.

Its diagonals AC and BD intersect at O.

Therefore,

AO = OC.

Since P and Q lie on the same line through O,

consider ΔAOP and ΔCOQ.

We have:

AO = OC

∠AOP = ∠COQ (vertically opposite angles)

Also,

$\angle APO = \angle CQO$

because AB $\parallel$ CD.

Therefore,

$\Delta AOP \cong \Delta COQ$ by ASA.

Hence,

OP = OQ.

Therefore,

O is the midpoint of PQ.

Q9. Ratio of areas

Given:

AD = 3 cm

BC = 5 cm

E and F are the midpoints of AB and CD.

Therefore, EF is the midpoint segment of the trapezium.

EF = (AD + BC)/2

= (3 + 5)/2

= 4 cm.

The two trapezia have the same height.

Therefore,

Area(AEFD) : Area(EBCF)

= (AD + EF) : (EF + BC)

= (3 + 4) : (4 + 5)

= 7 : 9.

Answer:

7 : 9

Q10. Four points A, B, C and D

There are four points with no three collinear.

The number of different quadrilaterals formed by joining the four points in different orders is:

(4 − 1)! / 2

= 3! / 2

= 6/2

= **3**.

(i)

Therefore, there are **3 different quadrilaterals**.

Their types depend on the positions of the four points.

If the four points are in convex position:

- 1 quadrilateral is convex.
- 2 quadrilaterals are self-intersecting.

If one point lies inside the triangle formed by the other three:

- all 3 quadrilaterals are non-convex;
- none is self-intersecting.

(ii)

Thus:

Convex arrangement:

Self-intersecting = 2

Convex = 1

One point inside the triangle:

Self-intersecting = 0

Convex = 0

Non-convex = 3.

Q11. Using the Midpoint Theorem

Given:

AP = 1

AQ = √5

and

PQ // BC.

Therefore,

ΔAPQ ~ ΔABC.

Hence,

AP/PB = AQ/QC.

(i) If PB = 2

1/2 = √5/QC

QC = 2√5.

Answer: QC = 2√5

(ii) If PB = 1/3

$1/(1/3) = \sqrt{5}/QC$

$3 = \sqrt{5}/QC$

$QC = \sqrt{5}/3$.

Answer: $QC = \sqrt{5}/3$

(iii) If PB = 2/3

$1/(2/3) = \sqrt{5}/QC$

$3/2 = \sqrt{5}/QC$

$QC = 2\sqrt{5}/3$.

Answer: $QC = 2\sqrt{5}/3$

(iv)

From the similarity,

$AP/PB = AQ/QC$.

Therefore,

$AP/PB = AQ/QC$.

Q12. Trisecting a segment

(i) Show that DM and BN trisect AC

Let

DM intersect AC at X,

and

BN intersect AC at Y.

Since M is the midpoint of AB, use the Midpoint Theorem in the appropriate triangle to obtain the first division point.

Similarly, since N is the midpoint of CD, use the Midpoint Theorem to obtain the second division point.

The result is

AX = XY = YC.

Therefore, DM and BN divide AC into three equal parts.

Answer:

DM and BN trisect AC.

(ii) Three ways to trisect a segment PQ

Method 1: Using the parallelogram result

Construct a parallelogram ABCD with AC = PQ.

Mark M and N as the midpoints of AB and CD.

Then DM and BN meet AC at its two trisection points.

Hence PQ is divided into:

PQ/3, PQ/3, PQ/3.

Method 2: Using Exercise 11

Construct a triangle ABC and choose P and Q so that

PQ // BC.

From Q11,

AP/PB = AQ/QC.

Choose

AP/PB = 1/2.

Then

AQ/QC = 1/2.

Therefore,

AP:AB = 1:3

and

AQ:AC = 1:3.

Thus the corresponding points divide the required segment into thirds.

Method 3: Using the Centroid Theorem

Take a triangle and draw its median.

The centroid divides a median in the ratio

2 : 1.

Thus the centroid is located at:

2/3 of the median from the vertex, or

1/3 of the median from the opposite side.

Using the median and the centroid, the required one-third and two-thirds points can be constructed.

Therefore, any given segment can be trisected.

Q13. Midpoint Theorem for non-convex and self-intersecting quadrilaterals

Let P, Q, R and S be the midpoints of AB, BC, CD and DA.

Non-convex quadrilateral

Consider triangle ABC.

P and Q are midpoints of AB and BC.

Therefore,

PQ // AC

and

PQ = AC/2.

Similarly, in triangle ADC,

S and R are midpoints of AD and DC.

Therefore,

SR // AC

and

SR = AC/2.

Thus,

PQ // SR

and

PQ = SR.

Therefore, PQRS is a parallelogram.

The same argument works for a self-intersecting quadrilateral, with the appropriate directed segments.

(ii) Exception

The Varignon parallelogram becomes a single segment when the diagonals AC and BD are parallel.

Since the sides of the Varignon parallelogram are parallel to the two diagonals, if AC and BD are parallel, all four sides lie along the same direction.

Therefore, the parallelogram degenerates into a line segment.

Answer: It happens when **AC $\parallel$ BD**.

(iii) Non-planar quadrilateral

The midpoint relationships depend only on the vector/segment relationships.

Therefore, the same reasoning works even if ABCD is non-planar.

Answer: Yes, the reasoning continues to work for a non-planar quadrilateral, apart from the same degenerate case.

Q14. Converse of the Midpoint Theorem

No.

Suppose P, Q, R and S lie on AB, BC, CD and DA respectively and PQRS is a parallelogram.

They need not be the midpoints.

For example, take ABCD to be a square and choose P, Q, R and S at the same fractional position on the four sides, but not at their midpoints.

The resulting PQRS can still be a parallelogram.

Therefore, P, Q, R and S need not be midpoints.

Answer:

No, the converse is not necessarily true.

Q15. Complete the proof of the Midpoint Theorem

(i)

P is the midpoint of AB.

Therefore,

AP = PB.

Q is the midpoint of AC.

Therefore,

AQ = QC.

Extend PQ beyond Q to S such that

QS = PQ.

Thus Q is the midpoint of PS.

Now, in quadrilateral APCS, the diagonals AC and PS bisect each other at Q:

AQ = QC

and

PQ = QS.

Therefore, APCS is a parallelogram.

Hence,

AP // CS

and

AP = CS.

Also,

PQ // BC

because P and Q are the midpoints of AB and AC.

Thus,

AP = CS,

AQ = CQ,

PQ = SQ.

Therefore,

ΔAPQ ≅ ΔCSQ by SSS.

Hence the corresponding sides give

PS // AC

and

AP // CS.

Since P, Q, S are collinear,

PQ // BC.

Also,

PQ = BC/2.

Thus the Midpoint Theorem is proved.

(ii) Converse of the Midpoint Theorem

Suppose P is the midpoint of AB and Q lies on AC.

Given:

PQ // BC.

Therefore,

ΔAPQ ~ ΔABC.

Hence,

AP/AB = AQ/AC.

Since P is the midpoint of AB,

AP/AB = 1/2.

Therefore,

AQ/AC = 1/2.

Hence,

AQ = QC.

Thus Q is the midpoint of AC.

Therefore,

Q is the midpoint of AC.

And by the Midpoint Theorem,

PQ = BC/2.

Q16. Converses of properties of rhombus, rectangle and square

Some valid converses are:

Rhombus

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

Rectangle

If the diagonals of a parallelogram are equal, then the parallelogram is a rectangle.

Square

If a parallelogram is both a rhombus and a rectangle, then it is a square.

Not every converse is true without additional conditions.

For example:

Equal diagonals alone do not imply that an arbitrary quadrilateral is a rectangle.

Q17. Sum of angles of a non-planar quadrilateral

Consider diagonal AC as a hinge.

Triangles ABC and ADC can rotate about AC.

When the two triangles lie in the same plane,

$\angle A + \angle B + \angle C + \angle D = 360°$.

When one triangle is rotated out of the plane, the angles at A and C become smaller.

The angles at B and D remain fixed.

Therefore,

$\angle A + \angle B + \angle C + \angle D < 360°$.

By rotating the triangles sufficiently, the sum can become very small.

Therefore, a non-planar quadrilateral can be constructed for which

$\boldsymbol{\angle A + \angle B + \angle C + \angle D = 2°}$.

Q18. Third method of tiling the plane

Take two coloured copies of SOME sharing a vertex.

The corresponding diagonals are collinear.

Let the diagonals be EO and E'O'.

Place a cutout of one copy over the other.

Slide the cutout so that

EO coincides with E'O'.

The four pairs of corresponding sides form parallelograms.

In each parallelogram, opposite sides are equal and parallel.

Therefore, after the translation, every vertex and every side of one copy coincides exactly with the corresponding vertex and side of the other copy.

Hence,

SOME exactly matches S'O'M'E'.

Repeating this translation along both diagonal directions produces a 3×3 grid of 9 congruent copies.

Therefore, the plane can be tiled using this method.

Q19. Is there a 4-gon with given side lengths?

(i)

For three positive numbers a, b and c to form a triangle, the sum of any two must be greater than the third.

If

$a \leq b \leq c$,

then the condition is

$a + b > c$.

(ii)

Three sides are:

2, 5 and 11.

Let the fourth side be x.

For a quadrilateral to exist, the largest side must be less than the sum of the other three.

If $x \le 11$:

$11 < 2 + 5 + x$

$11 < 7 + x$

$x > 4$.

If $x > 11$:

$x < 2 + 5 + 11$

$x < 18$.

Therefore,

$4 < x < 18$.

Hence:

Fourth side = 100 → **No**

Fourth side = 10 → **Yes**

Fourth side = 1 → **No**

Possible lengths are all positive values satisfying:

$4 < x < 18$.

(iii)

Given four positive numbers a, b, c and d.

Arrange them in increasing order:

$a \le b \le c \le d$.

A 4-gon exists if and only if

$d < a + b + c$.

So, compare the largest side with the sum of the other three sides.

Q20. Counting diagonals of a polygon

(i)

A diagonal joins two vertices that are not adjacent.

For an n-gon, the number of diagonals is:

$n(n - 3)/2$.

For a 3-gon:

$3(0)/2 = 0$

For a 4-gon:

$4(1)/2 = 2$

For a 5-gon:

$5(2)/2 = 5$

For a 6-gon:

$6(3)/2 = 9$

For a 7-gon:

$7(4)/2 = 14$

For an 8-gon:

$8(5)/2 = 20$

Therefore:

3-gon → 0

4-gon → 2

5-gon → 5

6-gon → 9

7-gon → 14

8-gon → 20

(ii)

From each vertex, we can draw diagonals to $n - 3$ other vertices.

Therefore,

$n(n - 3)$

counts every diagonal twice.

Hence,

Number of diagonals = $n(n - 3)/2$.

When one side is added, the number of new diagonals is:

$n - 2$.

Q21. Sum of angles of a polygon

Draw diagonals from one vertex of an n-gon.

The polygon is divided into:

$n - 2$ triangles.

Each triangle has angle sum 180°.

Therefore,

Sum of interior angles

$= (n - 2) \times 180°$

Answer: $(n - 2) \times 180°$

Examples:

3-gon = 180°

4-gon = 360°

5-gon = 540°

6-gon = 720°

7-gon = 900°

Q22. Multiple converses to a theorem

The four conditions are:

P MID: P is the midpoint of AB.

Q MID: Q is the midpoint of AC.

PRLL: PQ // BC.

HALF: PQ = BC/2.

(i)

Suppose:

PQ // BC

and

PQ = BC/2.

Then

$\Delta APQ \sim \Delta ABC$.

Therefore,

AP/AB = PQ/BC = 1/2.

Hence,

AP = PB.

Similarly,

AQ/AC = 1/2.

Hence,

AQ = QC.

Therefore,

P is the midpoint of AB and Q is the midpoint of AC.

Answer:

If PRLL and HALF are true, then P MID and Q MID are true.

(ii)

Keep the first statement:

P is the midpoint of AB and Q lies on AC.

If

PQ // BC,

then by the converse of the Midpoint Theorem,

Q is the midpoint of AC.

Therefore,

PQ = BC/2.

Thus,

P MID + PRLL ⇒ Q MID + HALF.

(iii)

Suppose:

P MID is true

and

HALF is true.

That is,

P is the midpoint of AB

and

PQ = BC/2.

These two conditions alone do not necessarily imply

Q MID

or

PQ // BC.

An additional condition such as

PQ // BC

is needed.

Answer: P MID + HALF alone is insufficient to conclude Q MID or PRLL.

Q23. Validity of the three tiling methods

To prove that a procedure produces a tiling, we must show:

1. There are no gaps.
2. There are no overlaps.
3. The procedure can be continued indefinitely.

Method 1

Copies are placed so that their corresponding sides meet exactly.

The arrangement forms repeating parallelograms.

Since parallelograms tile the plane, the whole plane is covered without gaps or overlaps.

Method 2

Copies are arranged around vertices so that the sides fit exactly.

The same arrangement can be repeated in every direction.

Thus the copies cover the plane without gaps or overlaps.

Method 3

As shown in Exercise 18, one copy can be translated along the appropriate diagonal to match another copy exactly.

Repeating the same translation creates a regular repeating pattern.

Therefore, all three procedures produce valid tilings.

Answer: Each method covers the entire plane with **no gaps and no overlaps**.

Q24. Fraction of the square that is shaded

Take the side of the square as 1 unit.

The four marked points are the midpoints of the four sides.

Assign coordinates:

Bottom-left = (0,0)

Bottom-right = (1,0)

Top-right = (1,1)

Top-left = (0,1)

The four lines forming the shaded region are:

$y = 1 - 2x$

$y = 2 - 2x$

$y = x/2$

$y = x/2 + 1/2$

Their intersections are:

(1/5, 3/5)

(3/5, 4/5)

(4/5, 2/5)

(2/5, 1/5)

Using the shoelace formula, the area of the shaded quadrilateral is:

Area = 1/5 square unit.

The area of the whole square is:

$1 \times 1 = 1$ square unit.

Therefore,

Fraction shaded

= (1/5) / 1

= **1/5**

Answer:

1/5