📘 Class 9 Maths – Ganita Manjari Part II

Chapter 13: Two Variables, One Line

End-of-Chapter Exercises – Complete Solutions

Q1. The graph of the line y = 3x, passing through (0, 0) and (2, 6), is given below.

(i) Identify the slope of the line from the graph and explain how you calculated it using the two points on the line.

The two points are:

(0, 0) and **(2, 6)**

Slope:

$m = (y_2 - y_1)/(x_2 - x_1)$

= (6 − 0)/(2 − 0)

= 6/2

= **3**

Answer:

Slope = 3

(ii) What is the y-intercept of the line? Explain its significance.

The line is:

y = 3x

Comparing with:

y = mx + c

we get:

c = 0

Therefore, the y-intercept is **0**.

The line intersects the y-axis at:

(0, 0)

Answer:

y-intercept = 0

It means that when **x = 0**, the value of y is **0**.

(iii) A water tank is being filled so that the water level y (in cm) after x minutes follows this graph.

The equation is:

y = 3x

(a) How high is the water after 5 minutes?

x = 5

y = 3(5)

y = 15 cm

Answer:

15 cm

(b) How many minutes will it take for the water to reach a height of 21 cm?

y = 21

21 = 3x

x = 21/3

x = 7 minutes

Answer:

7 minutes

(iv) Without drawing a new graph, determine whether the point (4, 10) lies on this line. Justify your answer.

Given:

y = 3x

For x = 4:

y = 3(4)

= 12

But the given point has y = 10.

Since:

10 ≠ 12

the point does not lie on the line.

Answer:

(4, 10) does not lie on the line.

(v) Plot the point where this line intersects the line x = 3. Explain how you found the coordinates.

Given:

x = 3

For the line:

y = 3x

y = 3(3)

= 9

Therefore, the point of intersection is:

Answer:

(3, 9)

Q2. In countries like the USA, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Celsius to Fahrenheit:

F = (9/5)C + 32

(i) Draw the graph of the linear equation above using Celsius on the x-axis and Fahrenheit on the y-axis.

Take some convenient values:

C (°C)	F (°F)
0	32
10	50
20	68
30	86
40	104

Plot:

(0, 32), (10, 50), (20, 68), (30, 86), (40, 104)

Joining these points gives a straight line.

(ii) If the temperature is 30°C, what is the temperature in Fahrenheit?

F = (9/5)(30) + 32

= 54 + 32

= **86°F**

Answer:

86°F

(iii) If the temperature is 95°F, what is the temperature in Celsius?

95 = (9/5)C + 32

95 − 32 = (9/5)C

63 = (9/5)C

$C = 63 \times 5/9$

$C = 7 \times 5$

$C = 35°C$

Answer:

35°C

(iv) If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?

When C = 0:

$F = (9/5)(0) + 32$

$F = 32°F$

When F = 0:

$0 = (9/5)C + 32$

$-32 = (9/5)C$

$C = -32 \times 5/9$

$C = -160/9°C$

$\approx -17.78°C$

Answer:

0°C = 32°F

0°F = −160/9°C ≈ −17.78°C

(v) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.

Let:

F = C

Using:

$F = (9/5)C + 32$

$C = (9/5)C + 32$

$C - (9/5)C = 32$

$-4/5\ C = 32$

$C = 32 \times (-5/4)$

$\mathbf{C = -40}$

Therefore:

F = −40°F

Answer:

Yes.

−40°C = −40°F

Q3. Solve the following system of equations graphically:

$\mathbf{2x + y = 6}$

$\mathbf{2x - y - 2 = 0}$

Rewrite the second equation:

$\mathbf{2x - y = 2}$

For the first line:

$2x + y = 6$

Some points:

- $x = 0 \rightarrow y = 6 \rightarrow$ **(0, 6)**
- $x = 3 \rightarrow y = 0 \rightarrow$ **(3, 0)**

For the second line:

$2x - y = 2$

- $x = 0 \rightarrow y = -2 \rightarrow$ **(0, −2)**
- $x = 1 \rightarrow y = 0 \rightarrow$ **(1, 0)**

When the two lines are plotted, they intersect at:

(2, 2)

Verification:

$2(2) + 2 = 6$ ✓

$2(2) - 2 - 2 = 0$ ✓

Answer:

$x = 2, y = 2$

Q4. Find the point of intersection of the lines shown on the cover page.

This question depends on the **specific graph on the textbook cover page**. The pages supplied here do not include that cover graph, so the coordinates cannot be determined reliably from the provided material.

Answer:

The cover-page graph is required to determine the point of intersection.

Q5. Give a formula to find the x-intercept of the line y = mx + c.

At the x-intercept:

$y = 0$

Therefore:

$0 = mx + c$

$mx = -c$

$x = -c/m$

provided **$m \neq 0$**.

Answer:

x-intercept = $-c/m$

Q6. A person is choosing between two mobile plans.

Plan A:

₹50 monthly fee + ₹0.20 per minute

Plan B:

₹30 monthly fee + ₹0.30 per minute

Let the number of minutes be **x**.

Cost of Plan A:

$A = 50 + 0.20x$

Cost of Plan B:

$B = 30 + 0.30x$

When is Plan A cheaper?

$50 + 0.20x < 30 + 0.30x$

$20 < 0.10x$

$x > 200$

Answer:

Plan A is cheaper for more than 200 minutes.

When is Plan B cheaper?

$30 + 0.30x < 50 + 0.20x$

$0.10x < 20$

$x < 200$

Answer:

Plan B is cheaper for fewer than 200 minutes.

When do both plans cost the same?

$50 + 0.20x = 30 + 0.30x$

$20 = 0.10x$

$x = \mathbf{200}$

Answer:

Both plans cost the same at **200 minutes**.

At 200 minutes:

Plan A = ₹50 + ₹40 = **₹90**

Plan B = ₹30 + ₹60 = **₹90**

Q7. How many lines exist that:

(i) have a given slope?

There are **infinitely many lines** with a given slope.

They are parallel to one another.

Answer:

Infinitely many

(ii) have a given slope and pass through a given point?

Through a given point, there is exactly **one line** with a given slope.

Answer:

Exactly one

Q8. For what values of p does the pair of equations have a unique solution?

Given:

$4x + py + 8 = 0$

$2x + 2y + 2 = 0$

For a unique solution:

$a_1/a_2 \neq b_1/b_2$

Therefore:

$4/2 \neq p/2$

$2 \neq p/2$

$p \neq 4$

Answer:

$p \neq 4$

Q9. Find the values of a and b for which the following system of equations has infinitely many solutions:

$(a + b)x - 2by = 5a + 2b + 1$

$3x - y = 14$

Write the first equation in standard form:

$(a + b)x - 2by - (5a + 2b + 1) = 0$

For infinitely many solutions:

$a_1/a_2 = b_1/b_2 = c_1/c_2$

Thus:

$(a + b)/3 = (-2b)/(-1) = [-(5a + 2b + 1)]/(-14)$

So:

$(a + b)/3 = 2b$

Therefore:

$a + b = 6b$

$a = 5b$ …(1)

Also:

$(5a + 2b + 1)/14 = 2b$

$5a + 2b + 1 = 28b$

$5a + 1 = 26b$

Using $a = 5b$:

$25b + 1 = 26b$

$b = 1$

Therefore:

$a = 5$

Answer:

$a = 5, b = 1$

Q10. Find the value of k for which the following system represents a pair of coincident lines:

$x + 2y = 3$

$(k - 1)x + (k + 1)y = k + 3$

For coincident lines, the second equation must be a multiple of the first.

From the x-coefficients:

$k - 1 = \lambda$

From the y-coefficients:

$k + 1 = 2\lambda$

Therefore:

$k + 1 = 2(k - 1)$

$k + 1 = 2k - 2$

$k = 3$

Check:

$k = 3$ gives:

Second equation:

$2x + 4y = 6$

which is exactly twice:

$x + 2y = 3$.

Answer:

k = 3

Q11.

(i) Robot 1 starts from the origin and traces a path by repeatedly moving 3 units to the right and then 4 units upward. Robot 2 starts from the point (10, 0) and repeatedly moves 1 unit to the right and then 2 units upward. Will the paths traced by these two robots intersect and if so, where?

Robot 1 follows the overall line:

Rise = 4

Run = 3

So its path has slope:

4/3

Since it starts at (0, 0):

$y = (4/3)x$

Robot 2 starts at (10, 0):

Rise = 2

Run = 1

So:

$y = 2(x - 10)$

At the intersection:

$(4/3)x = 2(x - 10)$

$4x = 6x - 60$

$2x = 60$

$x = 30$

$y = (4/3)(30)$

$y = 40$

Therefore, the paths intersect at:

Answer:

(30, 40)

(ii) Robot 1 starts from (3, 0) and traces a path by repeatedly moving 5 units to the right and then 5 units downward. Robot 2 starts from (7, 0) and repeatedly moves 5 units to the right and then 3 units downwards.

Will the paths traced by these two robots intersect and if so, where?

Robot 1 has slope:

$$-5/5 = -1$$

Equation through (3, 0):

$$y = -(x - 3)$$

or

$$y = -x + 3$$

Robot 2 has slope:

$$-3/5$$

Equation through (7, 0):

$$y = -(3/5)(x - 7)$$

or

$$y = -3x/5 + 21/5$$

Equating:

$-x + 3 = -3x/5 + 21/5$

Multiply by 5:

$-5x + 15 = -3x + 21$

$-2x = 6$

$x = -3$

Then:

$y = -(-3) + 3$

$= 6$

The mathematical intersection of the extended lines is **(−3, 6)**.

However, both robots move only to the **right** from their starting points, so their actual traced paths have x-coordinates at least 3 and 7 respectively.

Therefore, **(−3, 6) is not on either actual traced path**.

Answer:

The paths do not intersect.

Q12. At a certain time, Jacob notices that his digital watch reads ‘a’ minutes after two o’clock. Fifteen minutes later, it reads ‘b’ minutes after three o’clock. He noticed that a is six times greater than b. What time was it when he looked at his watch for the second time?

Let the first time be:

2:a

Fifteen minutes later, the time is:

3:b

From 2:a to 3:b:

60 + b minutes pass.

But this is also:

a + 15 minutes.

Therefore:

a + 15 = 60 + b

$a - b = 45$ …(1)

Given:

$a = 6b$ …(2)

Substitute:

$6b - b = 45$

$5b = 45$

$b = 9$

Therefore:

$a = 6(9)$

$a = 54$

The first time was:

2:54

Fifteen minutes later:

3:09

Answer:

3:09

Q13. The sum of the digits of a two-digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. Find the number.

Let the tens digit be x and the units digit be y.

Then:

$x + y = 15$ …(1)

Original number:

$10x + y$

Reversed number:

$10y + x$

Given:

$10y + x = 10x + y + 9$

$9y - 9x = 9$

$y - x = 1$ …(2)

Add (1) and (2):

$2y = 16$

$y = 8$

Therefore:

$x = 7$

The number is:

Answer:

78

Q14. In a cyclic quadrilateral ABCD, $\angle A = (x + 7)°$, $\angle B = (y + 8)°$, $\angle C = (3y + 23)°$ and $\angle D = (4x + 12)°$. Find all four angles of the cyclic quadrilateral.

In a cyclic quadrilateral, opposite angles are supplementary.

Therefore:

$\angle A + \angle C = 180°$

$(x + 7) + (3y + 23) = 180$

$x + 3y = 150$ …(1)

Also:

$\angle B + \angle D = 180°$

$(y + 8) + (4x + 12) = 180$

$4x + y = 160$ …(2)

From (1):

$x = 150 - 3y$

Substitute in (2):

$4(150 - 3y) + y = 160$

$600 - 12y + y = 160$

$-11y = -440$

$y = 40$

Therefore:

$x = 150 - 120$

$x = 30$

Now find the angles:

$\angle A = x + 7 = 30 + 7 =$ **$37°$**

$\angle B = y + 8 = 40 + 8 =$ **$48°$**

$\angle C = 3y + 23 = 120 + 23 =$ **$143°$**

$\angle D = 4x + 12 = 120 + 12 =$ **$132°$**

Answer:

$\angle$**A = 37°**

$\angle$**B = 48°**

$\angle$**C = 143°**

$\angle$**D = 132°**

Check:

37 + 48 + 143 + 132 = **360°** ✓

Q15. A train moving with uniform speed for a certain distance takes 6 hours less if its speed is increased by 6 km/hour. It would have taken 6 hours more had its speed been decreased by 4 km/hour. Find the distance travelled and the speed of the train.

Let:

Original speed = **v km/h**

Original time = **t hours**

Distance:

D = vt

First condition

When speed is increased by 6:

New speed = v + 6

New time = $t - 6$

Therefore:

$$vt = (v + 6)(t - 6)$$

Expand:

$vt = vt - 6v + 6t - 36$

$6v = 6t - 36$

$v = t - 6$ …(1)

Second condition

When speed is decreased by 4:

New speed = $v - 4$

New time = $t + 6$

Therefore:

$$vt = (v - 4)(t + 6)$$

Expand:

$vt = vt + 6v - 4t - 24$

$6v = 4t + 24$ …(2)

From (1):

$t = v + 6$

Substitute in (2):

$6v = 4(v + 6) + 24$

$6v = 4v + 24 + 24$

$6v = 4v + 48$

$2v = 48$

$v = 24$

Therefore:

$t = 24 + 6$

$t = 30$

Distance:

$D = vt$

$= 24 \times 30$

= **720 km**

Answer:

- **Speed = 24 km/h**
- **Time = 30 hours**
- **Distance = 720 km**

Q16. The age of a father is equal to the sum of the ages of his four children. After 20 years, the sum of the ages of the children will be twice the age of the father. Find the age of the father.

Let the present age of the father be **x years**.

Therefore, the present sum of the ages of the four children is also:

x years

After 20 years:

Father's age:

x + 20

Each of the four children becomes 20 years older, so the total age of the children increases by:

$4 \times 20 = 80$ years

Therefore, the sum of children's ages after 20 years is:

$x + 80$

According to the question:

$x + 80 = 2(x + 20)$

$x + 80 = 2x + 40$

$80 - 40 = 2x - x$

$x = 40$

Answer:

The father's present age is 40 years.