📘 Class 9 Maths – Ganita Manjari Part II

Chapter 14: Math of Space: Surface Area and Volume

End-of-Chapter Exercises – Complete Solutions

Note: Questions marked with * and the guesstimate/project questions are intended to involve estimation and assumptions. For those, the numerical answer can vary depending on the assumptions made. The textbook itself emphasises making a guess first for guesstimate problems.

Q1. Estimate how many scoops of ice cream can be obtained from a cuboidal container of dimensions 10 cm × 15 cm × 20 cm.

Step 1: Make an estimate

Suppose one scoop of ice cream occupies approximately **100 cm³**.

Volume of container:

V = 10 × 15 × 20 = 3000 cm³

Number of scoops:

3000 ÷ 100 = 30

✅ Answer:

Approximately 30 scoops

The answer is an estimate because the actual size of a scoop can vary.

Q2. A cube of integer side length a is made of unit cubes. Write an expression giving the number of unit cubes to be added to make a cube of side length a + 1.

Original number of unit cubes:

a^3

New cube:

$(a + 1)^3$

Number of cubes to be added:

$(a + 1)^3 - a^3$

Expand:

$= a^3 + 3a^2 + 3a + 1 - a^3$

$=$ **$3a^2 + 3a + 1$**

✅ Answer:

$3a^2 + 3a + 1$ unit cubes

Q3. Solve the following

(i) Could a person drink enough water in a lifetime to fill an entire room the size of your classroom?

This is a **guesstimate**, so we need reasonable assumptions.

Suppose:

- A person drinks about **2 litres of water per day**.
- Lifetime ≈ **80 years**.
- Classroom dimensions ≈ **8 m × 6 m × 3 m**.

Water consumed in a lifetime

80 years ≈ 80 × 365 = 29,200 days

Water:

29,200 × 2 = 58,400 litres

Since:

1000 litres = 1 m^3

Volume ≈ **58.4 m^3**

Classroom volume:

8 × 6 × 3 = 144 m^3

Therefore:

58.4 m^3 < 144 m^3

✅ Answer:

Under these assumptions, **No**. A person would drink about **58 m^3** of water in a lifetime, whereas the classroom would have a volume of about **144 m^3**.

The result depends on the assumptions made.

(ii) Estimate the number of bricks used to build the walls of your classroom.

Suppose:

- Classroom = **8 m × 6 m × 3 m**
- Wall thickness ≈ **0.2 m**

- Brick dimensions ≈ **19 cm × 9 cm × 9 cm**

Approximate volume of four walls:

Perimeter:

2(8 + 6) = 28 m

Wall volume:

$28 \times 3 \times 0.2 = 16.8\ m^3$

Volume of one brick:

$0.19 \times 0.09 \times 0.09$

$\approx 0.00154\ m^3$

Number of bricks:

$16.8 \div 0.00154 \approx 10{,}900$

Allowing for doors, windows and mortar, this is roughly:

✅ Answer:

About 10,000–11,000 bricks

This is only an estimate.

Q4. You are given two options: (A) one chocolate cube of side 50 mm, (B) hundred chocolate cubes of side 10 mm. Which option gives you more chocolate?

Option A

Side = 50 mm

Volume:

$50^3 = 125,000\ mm^3$

Option B

Volume of one cube:

$10^3 = 1000\ mm^3$

For 100 cubes:

$100 \times 1000 = 100,000\ mm^3$

Compare:

$125,000 > 100,000$

Difference:

$125,000 - 100,000$

= **$25,000\ mm^3$**

✅ Answer:

Option A gives more chocolate.

Q5. If all the people in the world are crowded together in one location, how much area would it cover?

This is a guesstimate.

Suppose the world population is approximately:

8.3 billion people

Assume each person occupies about:

$0.25\ m^2$

Then:

Area:

$8.3 \times 10^9 \times 0.25$

= **$2.075 \times 10^9\ m^2$**

Since:

$1\ km^2 = 10^6\ m^2$

Area:

$2075\ km^2$

✅ Answer:

Approximately **$2{,}100\ km^2$**.

The actual estimate depends on how closely the people are packed.

Q6. A school provides milk to its students in cylindrical glasses, each with a diameter of 7 cm. If each glass is filled with milk to a height of 12 cm, how many litres of milk are needed to serve 1600 students?

Diameter:

7 cm

Radius:

$r = 3.5$ cm

Height:

$h = 12$ cm

Volume of one glass:

$V = \pi r^2 h$

Using $\pi = 22/7$:

$V = 22/7 \times 3.5^2 \times 12$

$= 22/7 \times 12.25 \times 12$

= **$462\ cm^3$**

For 1600 students:

$462 \times 1600 = 739{,}200\ cm^3$

Since:

$1000\ cm^3 = 1$ litre

Therefore:

$739{,}200 \div 1000 = 739.2$ litres

✅ Answer:

739.2 litres of milk

Q7. The surface area of a sphere of radius 5 cm is five times the area of the curved surface of a cone of radius 4 cm. Find the height and volume of the cone.

Surface area of sphere:

$4\pi r^2$

= $4\pi \times 5^2$

= **100π**

This is five times the curved surface area of the cone.

Therefore:

CSA of cone = $100\pi/5$

= **20π**

For cone:

CSA = πrl

Therefore:

$\pi \times 4 \times l = 20\pi$

$4l = 20$

$l = 5$ cm

Now:

$l^2 = r^2 + h^2$

$5^2 = 4^2 + h^2$

$25 = 16 + h^2$

$h^2 = 9$

$h = 3$ cm

Volume of cone

$V = 1/3\ \pi r^2 h$

= $1/3 \times \pi \times 16 \times 3$

= **16π cm^3**

Using $\pi = 22/7$:

$V = 352/7 \approx 50.29\ cm^3$

✅ **Answers:**

Height = 3 cm

Volume = $16\pi\ cm^3 \approx 50.29\ cm^3$

Q8. Take Earth to be a perfect sphere with radius 6370 km. Take Jupiter to be a perfect sphere with radius 69,900 km. Take the Sun to be a perfect sphere with radius 6,95,700 km. Compute approximately:

(i) Ratio of the volume of Jupiter to the volume of Earth

Volume of a sphere:

$V = 4/3\ \pi r^3$

Therefore:

$VJ/VE = (69{,}900/6{,}370)^3$

≈ 1321.34

✅ **Answer:**

Volume of Jupiter : Volume of Earth ≈ 1321 : 1

(ii) Ratio of the volume of the Sun to the volume of Earth

$VS/VE = (6{,}95{,}700/6{,}370)^3$

$\approx$ 1,302,710

✅ Answer:

Volume of Sun : Volume of Earth $\approx$ 1,302,710 : 1

Q9. Show that the volume of a sphere is equal to 2/3 of the volume of the smallest cylinder which encloses it.

Let the sphere have radius **r**.

The smallest cylinder enclosing the sphere has:

- Radius = **r**
- Height = **2r**

Volume of sphere

$VS = 4/3\ \pi r^3$

Volume of cylinder

$VC = \pi r^2(2r)$

= **$2\pi r^3$**

Therefore:

$VS/VC = (4/3\ \pi r^3)/(2\pi r^3)$

= **2/3**

Hence:

✅ Answer:

Volume of sphere = 2/3 × volume of the smallest enclosing cylinder.

Q10. Suppose the Earth is perfectly spherical. A string is wrapped tightly around the equator. Another string, 1 metre longer, is placed around the Earth so that it forms a larger circle, staying the same distance above the ground everywhere. How high above the ground is the second string?

Let the original radius be **R**.

Original circumference:

2πR

New circumference:

2πR + 1

Let the increase in radius be **h**.

Then:

$2\pi(R + h) = 2\pi R + 1$

$2\pi R + 2\pi h = 2\pi R + 1$

$2\pi h = 1$

Therefore:

$h = 1/(2\pi)$

≈ 0.159 m

$= 15.9$ cm

✅ Answer:

Approximately 15.9 cm above the ground.

For the Moon, Jupiter and a volleyball:

The result is the same because the radius cancels out.

Moon: ≈ 15.9 cm

Jupiter: ≈ 15.9 cm

Volleyball: ≈ 15.9 cm

Why?

Because the increase in circumference is:

2π × increase in radius

It does not depend on the original radius.

Q11. We have a cylinder with base radius r cm and height h cm. A square pyramid fits inside it. The square base of the pyramid lies on the base of the cylinder, its corners on the boundary of the cylinder. The apex of the pyramid lies on the top of the cylinder. Find the ratio of the volume of this pyramid to the volume of the cylinder.

The square is inscribed in the circular base of the cylinder.

The diagonal of the square equals the diameter of the circle:

Diagonal = 2r

For a square:

Area = $\text{diagonal}^2/2$

Therefore:

Area of square:

= $(2r)^2/2$

= $\mathbf{2r^2}$

Volume of pyramid

VP = 1/3 × base area × height

= $1/3 \times 2r^2 \times h$

= $\mathbf{2r^2h/3}$

Volume of cylinder

VC = $\pi r^2 h$

Therefore:

VP : VC

= **$2/(3\pi)$**

✅ Answer:

Volume of pyramid : Volume of cylinder = 2 : 3π

Q12. What is the change in volume when:

(i) The length of a cuboid with dimensions l, w, h is increased by 1 unit?

Original volume:

$V = lwh$

New volume:

$V' = (l + 1)wh$

$= lwh + wh$

Change:

$V' - V = wh$

✅ **Answer:**

wh cubic units

(ii) The radius of a cylinder with dimensions r, h is increased by 1 unit?

Original volume:

$V = \pi r^2 h$

New radius:

$r + 1$

New volume:

$V' = \pi(r + 1)^2 h$

$= \pi(r^2 + 2r + 1)h$

$= \pi r^2 h + 2\pi rh + \pi h$

Change:

$= 2\pi rh + \pi h$

✅ **Answer:**

$2\pi rh + \pi h$ cubic units

(iii) The radius of a sphere is decreased by 1 unit?

Original radius = **r**

New radius = **r − 1**

Original volume:

$V = 4/3\ \pi r^3$

New volume:

$V' = 4/3\ \pi(r - 1)^3$

Decrease in volume:

$V - V'$

$= 4/3\pi[r^3 - (r - 1)^3]$

Now:

$(r - 1)^3 = r^3 - 3r^2 + 3r - 1$

Therefore:

$r^3 - (r - 1)^3$

= **$3r^2 - 3r + 1$**

Hence decrease:

$4/3\ \pi(3r^2 - 3r + 1)$

✅ Answer:

Decrease = $4/3\ \pi(3r^2 - 3r + 1)$ cubic units

Q13. Given a cube with volume V, express its total surface area S in terms of V.

Let the side of cube be **a**.

Volume:

$V = a^3$

Therefore:

$a = V^{(1/3)}$

Surface area:

$S = 6a^2$

Therefore:

$S = 6V^{(2/3)}$

✅ Answer:

$S = 6V^{(2/3)}$

Q14. Given a cube with total surface area S, express its volume V in terms of S.

For a cube:

$S = 6a^2$

Therefore:

$a^2 = S/6$

So:

$a = \sqrt{(S/6)}$

Volume:

$V = a^3$

Therefore:

$V = (S/6)^{(3/2)}$

✅ **Answer:**

$V = (S/6)^{(3/2)}$

Q15. A cylindrical glass of height 25 cm and radius 4 cm has water up to a height of 16 cm. A crow wants to drink water from this glass. The water must be at a height of 20 cm. How many marbles of radius 1 cm should the crow drop into the glass?

Increase in water level:

$20 - 16 = 4$ cm

Volume that must be displaced:

$V = \pi r^2 h$

$= \pi \times 4^2 \times 4$

$=$ **64π cm^3**

Volume of one marble:

$V = 4/3\pi(1)^3$

$=$ **$4\pi/3$ cm^3**

Number of marbles:

$64\pi \div (4\pi/3)$

$= 64 \times 3/4$

= **48**

✅ Answer:

48 marbles

Q16. Find the volume of ink in a new ball point pen. Take the necessary measurements and make approximations, as needed.

This is an **activity/guesstimate question**, so there is no single fixed numerical answer.

For example, suppose the ink chamber is approximately cylindrical with:

- radius = **1.5 mm = 0.15 cm**
- length = **12 cm**

Then:

$V = \pi r^2 h$

$= \pi \times (0.15)^2 \times 12$

$= 0.27\pi$

$\approx 0.85\ cm^3$

Since:

$1\ cm^3 = 1\ mL$

✅ Sample estimate:

Approximately 0.85 mL of ink

Your answer may differ depending on the measurements taken.

Q17. Solve

(i) A ball of chapati dough of radius 6 cm is prepared. Estimate how many chapatis can be made from it?

Volume of dough ball:

$V = 4/3\pi r^3$

$= 4/3 \times \pi \times 6^3$

$=$ **288π cm^3**

≈ 905 cm^3

Now make an estimate for one chapati.

Suppose one chapati has approximately:

- radius = **8 cm**
- average thickness = **0.3 cm**

Volume of one chapati:

$V \approx \pi r^2 h$

$= \pi \times 8^2 \times 0.3$

$=$ **19.2π**

≈ 60.3 cm^3

Number of chapatis:

$905 \div 60.3 \approx 15$

✅ Sample estimate:

Approximately 15 chapatis

The actual answer depends on the size and thickness of the chapatis.

(ii) Cut a coconut/muskmelon in half and find out the approximate volume of edible coconut flesh/fruit by taking the necessary measurements.

This is a measurement-based activity.

For example, approximate the edible part as a sphere/hemisphere or another suitable shape.

If the edible portion is modelled as a sphere of radius **r**:

$V \approx 4/3\pi r^3$

Measure the fruit and substitute the measured value of **r**.

✅ Answer:

No single numerical answer is fixed; it depends on the fruit measured and the assumptions made.

Q18. Looking at the Ganita Manjari, Grade 9, Part 2 textbook, Sheela wonders:

(i) If all the pages of this textbook were laid out side-by-side on the floor would they cover the entire classroom floor?

This is a guesstimate.

Suppose:

- Number of pages ≈ **168**
- Page dimensions ≈ **21 cm × 28 cm**

Area of one page:

$21 \times 28 = 588$ cm²

Total area:

$168 \times 588 = 98{,}784\ cm^2$

Convert to m^2:

$98{,}784 \div 10{,}000 \approx 9.88\ m^2$

A typical classroom may have an area of about:

$8\ m \times 6\ m = 48\ m^2$

Therefore:

$9.88\ m^2 < 48\ m^2$

✅ Sample answer:

No, the pages would not cover the entire classroom floor.

They would cover roughly **$10\ m^2$**, under these assumptions.

(ii) What is the maximum number of textbooks that can fit in an empty storeroom of dimensions 15 ft × 20 ft × 30 ft?

Volume of storeroom:

$15 \times 20 \times 30 = 9000\ ft^3$

This is a guesstimate because the thickness and packing arrangement of the textbook must be estimated.

Suppose one textbook occupies approximately:

$0.9\ ft \times 0.7\ ft \times 0.05\ ft$

Volume:

$0.0315\ ft^3$

Maximum theoretical number:

$9000 \div 0.0315 \approx 285{,}714$

✅ Sample estimate:

About 2.9 lakh textbooks

In actual storage, the number would be lower because of gaps and packing arrangements.

Q19. If the entire human population decided to climb into one giant cube, how long would the side have to be?

This is a guesstimate.

Suppose:

- World population ≈ **8.3 billion**
- Volume occupied by one person when tightly packed ≈ **$0.07\ m^3$**

Total volume:

$8.3 \times 10^9 \times 0.07$

= **$581 \times 10^6\ m^3$**

Let side of cube be **a**.

$a^3 = 581 \times 10^6$

Therefore:

$a \approx 834$ m

✅ Sample answer:

The cube would have a side of approximately 830–840 m.

The answer changes depending on the assumed volume per person.

Q20. Earth's surface has an estimated volume of 1.38 billion km^3 of water. Suppose the Earth is a perfect sphere and all this water forms a uniform layer completely covering Earth's surface. Earth's radius is approximately 6371 km.

(i) Write an expression that gives the thickness of this water layer.

Let the thickness of the water layer be **t km**.

Volume of the water layer:

$$V = 4/3\pi[(R + t)^3 - R^3]$$

Given:

$$V = 1.38 \times 10^9 \text{ km}^3$$

and

$$R = 6371 \text{ km}$$

Therefore:

$$1.38 \times 10^9 = 4/3\pi[(6371 + t)^3 - 6371^3]$$

This is the required expression.

For a thin layer, we can approximate:

$$V \approx 4\pi R^2 t$$

Therefore:

$$t \approx 1.38 \times 10^9 / (4\pi \times 6371^2)$$

(ii) Simplify the expression using a calculator.

Substituting:

$t \approx 1.38 \times 10^9 / [4\pi(6371)^2]$

gives:

$t \approx 2.71$ km

✅ Answer:

Thickness of water layer ≈ 2.7 km

⭐ Q21. Give the dimension of a cuboid whose volume is halved when its surface area is doubled.

The statement does **not specify what change is being made to the cuboid's dimensions**. Therefore, there is not enough information to determine a unique set of dimensions.

For example, if the dimensions are allowed to change arbitrarily from one cuboid to another, many pairs of cuboids can satisfy:

$V_2 = V_1/2$

and

$SA_2 = 2SA_1$

So a unique answer cannot be obtained from the wording alone.

✅ Answer:

The question is underdetermined as stated; an additional condition about how the dimensions change is required.

Q22. Project: Find the volume of your house making necessary approximations. Present how you solved it.

This is a project question.

A suitable method is to divide the house into simple solids.

For example:

Volume of house = Volume of main cuboid + Volume of additional rooms + Volume of other simple shapes

Suppose a house is approximately:

$12 \text{ m} \times 8 \text{ m} \times 3 \text{ m}$

Then:

$V = 12 \times 8 \times 3$

$=$ **288 m^3**

If there is another room of dimensions:

$4 \text{ m} \times 3 \text{ m} \times 3 \text{ m}$

its volume is:

36 m^3

Total:

$288 + 36 = 324 \text{ m}^3$

✅ Sample answer:

Approximate volume = 324 m^3

For an actual project, measure your own house and divide it into cuboids or other simple shapes before adding their volumes.