

Exercise 2.2 Class 10 Maths CH- 2

Polynomials Solutions

(From New NCERT book) Session
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Exercise 2.2 Page: 23

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and their coefficients:

- (i) $x^2 - 2x - 8$ (ii) $4s^2 - 4s + 1$ (iii) $6x^2 - 3 - 7x$ (iv) $4u^2 + 8u$
(v) $t^2 - 15$ (vi) $3x^2 - x - 4$

Solution:

[Concept: First factorise the given polynomial and then put each factor equal to zero to find the required Zeroes and then for verification show that

Sum of zeroes = - Coefficient of x / Coefficient of x^2

Product of zeroes = Constant term / Coefficient of x^2

(i) Given: $x^2 - 2x - 8$

To Find: The zeroes of the quadratic polynomial $x^2 - 2x - 8$, and verify the relationship between the zeroes and the coefficients

Solution:

Let $f(x) = x^2 - 2x - 8$

By splitting the middle term, we get

$$f(x) = x^2 - 4x + 2x - 8$$

$$[-2 = -4 + 2 \text{ and } -4 \times 2 = -8]$$

$$f(x) = x(x - 4) + 2(x - 4)$$

$$f(x) = (x + 2)(x - 4)$$

On putting $f(x) = 0$, we get

$$(x + 2)(x - 4) = 0$$

$$x + 2 = 0 \text{ or } x - 4 = 0$$

$$x = -2 \text{ or } x = 4$$

Hence, the zeroes of the given polynomial are - 2 and 4 i.e $\alpha = -2$ and $\beta = 4$

Verification:

Zeroes are -2 and 4.

$$\alpha = -2 \text{ and } \beta = 4$$

$$\therefore \alpha + \beta = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$-2 + 4 = -(-2)$$

$$2 = 2 \text{ verified.}$$

$$\therefore \alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$(-2)(4) = \frac{-8}{1} = -8$$

$$-8 = -8 \text{ verified.}$$

So, the relationship between the zeroes and the coefficients is verified.

(ii) Given: $4s^2 - 4s + 1$

To Find: The zeroes of the quadratic polynomial $4s^2 - 4s + 1$, and verify the relationship between the zeroes and the coefficients

Solution:

$$\text{Let } f(s) = 4s^2 - 4s + 1$$

By splitting the middle term, we get

$$f(s) = 4s^2 - 2s - 2s + 1$$

$$[-4 = -2 - 2 \text{ and } -2 \times -2 = 4]$$

$$f(s) = 2s(2s - 1) - 2s(2s - 1)$$

$$f(s) = (2s - 1)(2s - 1)$$

On putting $f(s) = 0$, we get

$$(2s - 1)(2s - 1) = 0$$

$$2s - 1 = 0 \text{ or } 2s - 1 = 0$$

$$s = 1/2 \text{ or } s = 1/2$$

Hence, the zeroes of the given polynomial are $1/2$ and $1/2$ i.e $\alpha = 1/2$ and $\beta = 1/2$.

Verification:

$$\alpha = \frac{1}{2}, \beta = \frac{1}{2}$$

$$\therefore \alpha + \beta = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\frac{1}{2} + \frac{1}{2} = -\frac{-4}{4}$$

$$1 = -(-1) = 1 \text{ verified.}$$

$$\alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$$

$$\frac{1}{4} = \frac{1}{4} \text{ verified.}$$

So, the relationship between the zeroes and the coefficients is verified.

(iii) Given: $6x^2 - 3 - 7x$

To Find: The zeroes of the quadratic polynomial $6x^2 - 3 - 7x$, and verify the relationship between the zeroes and the coefficients

Solution:

Rearrange the given quadratic polynomial

$$\text{Let } f(x) = 6x^2 - 7x - 3$$

By splitting the middle term, we get

$$f(x) = 6x^2 - 9x + 2x - 3$$

$$[-7 = -9 + 2 \text{ and } -9 \times 2 = 18]$$

$$f(x) = 3x(2x - 3) + 1(2x - 3)$$

$$f(x) = (3x + 1)(2x - 3)$$

On putting $f(x) = 0$, we get

$$(3x + 1)(2x - 3) = 0$$

$$3x + 1 = 0 \text{ or } 2x - 3 = 0$$

$$x = -1/3 \text{ or } x = 3/2$$

Hence, the zeroes of the given polynomial are $-1/3$ and $3/2$ i.e

$$\alpha = -1/3 \text{ and } \beta = 3/2$$

Verification:

$$\alpha = -\frac{1}{3}, \beta = \frac{3}{2}$$

$$\begin{aligned} \therefore \alpha + \beta &= -\frac{1}{3} + \frac{3}{2} = \frac{-2+9}{6} = \frac{7}{6} \\ &= \frac{-(-7)}{6} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} \end{aligned}$$

$$\begin{aligned} \text{and } \alpha\beta &= \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} \\ &= \frac{\text{constant term}}{\text{coefficient of } x^2} \text{ verified.} \end{aligned}$$

So, the relationship between the zeroes and the coefficients is verified.

(iv) Given: $4u^2 + 8u$

To Find: The zeroes of the quadratic polynomial $4u^2 + 8u$ and verify the relationship between the zeroes and the coefficients

Solution:

$$\text{Let } f(u) = 4u^2 + 8u$$

$$f(u) = 4u(u + 2)$$

On putting $f(u) = 0$, we get

$$4u(u + 2) = 0$$

$$4u = 0 \text{ or } u + 2 = 0$$

$$u = 0 \text{ or } u = -2$$

Hence, the zeroes of the given polynomial are 0 and -2 i.e $\alpha = 0$ and $\beta = -2$.

Verification:

$$\alpha = 0, \beta = -2$$

$$\begin{aligned}\therefore \alpha + \beta &= 0 + (-2) = -2 = -\frac{8}{4} \\ &= -\frac{\text{coefficient of } u}{\text{coefficient of } u^2}\end{aligned}$$

$$\begin{aligned}\text{and } \alpha\beta &= 0(-2) = \frac{0}{4} \\ &= \frac{\text{constant term}}{\text{coefficient of } u^2} \text{ verified.}\end{aligned}$$

So, the relationship between the zeroes and the coefficients is verified.

(v) Given: $t^2 - 15$

To Find: The zeroes of the quadratic polynomial $t^2 - 15$ and verify the relationship between the zeroes and the coefficients

Solution:

$$\text{Let } f(t) = t^2 - 15$$

$$f(t) = t^2 - (\sqrt{15})^2$$

$$f(t) = (t - \sqrt{15})(t + \sqrt{15})$$

$$[a^2 - b^2 = (a - b)(a + b)]$$

On putting $f(t) = 0$, we get

$$(t - \sqrt{15})(t + \sqrt{15}) = 0$$

$$t - \sqrt{15} = 0 \text{ or } t + \sqrt{15} = 0$$

$$t = \sqrt{15} \text{ or } t = -\sqrt{15}$$

**Hence, the zeroes of the given polynomial are $\sqrt{15}$ and $-\sqrt{15}$
i.e $\alpha = \sqrt{15}$ and $\beta = -\sqrt{15}$.**

Verification:

$$\begin{aligned}\alpha &= -\sqrt{15}, \beta = \sqrt{15} \\ \therefore \alpha + \beta &= -\sqrt{15} + \sqrt{15} = 0 = -\frac{0}{1} \\ &= \frac{-\text{coefficient of } t}{\text{coefficient of } t^2} \text{ verified.} \\ \text{and } \alpha\beta &= (-\sqrt{15})(\sqrt{15}) \\ &= -15 = \frac{-15}{1} \\ &= \frac{\text{constant term}}{\text{coefficient of } t^2} \text{ verified.}\end{aligned}$$

So, the relationship between the zeroes and the coefficients is verified.

(vi) Given: $3x^2 - x - 4$

To Find: The zeroes of the quadratic polynomial $3x^2 - x - 4$
and verify the relationship between the zeroes and the coefficients

Solution:

$$\text{Let } f(x) = 3x^2 - x - 4$$

By splitting the middle term, we get

$$f(x) = 3x^2 - 4x + 3x - 4$$

$$[-1 = -4 + 3 \text{ and } -4 \times 3 = -12]$$

$$f(x) = x(3x - 4) + 1(3x - 4)$$

$$f(x) = (3x - 4)(x + 1)$$

On putting $f(x) = 0$, we get

$$(3x - 4)(x + 1) = 0$$

$$3x - 4 = 0 \text{ or } x + 1 = 0$$

$$x = 4/3 \text{ or } x = -1$$

Hence, the zeroes of the given polynomial are $\frac{4}{3}$ and -1 i.e $\alpha = \frac{4}{3}$ and $\beta = -1$.

Verification:

$$\alpha = \frac{4}{3}, \beta = -1$$

$$\begin{aligned}\therefore \alpha + \beta &= \frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(-1)}{3} \\ &= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} \text{ verified.} \\ \alpha\beta &= \frac{4}{3}(-1) = \frac{-4}{3} \\ &= \frac{\text{constant term}}{\text{coefficient of } x^2} \text{ verified.}\end{aligned}$$

So, the relationship between the zeroes and the coefficients is verified.

2. Find a quadratic polynomial, each with the given numbers as the sum and product of zeroes, respectively:

(i) $\frac{1}{4}, -1$	(ii) $\sqrt{2}, \frac{1}{3}$	(iii) $0, \sqrt{5}$
(iv) $1, 1$	(v) $\frac{-1}{4}, \frac{1}{4}$	(vi) $4, 1$

Solution:

[Concept - Put the sum and product of zeros in $x^2 - (\text{Sum of zeroes})x + \text{Product of zeroes}$ to get the required quadratic polynomial.]

(i) $\frac{1}{4}, -1$

Given: Sum of zeroes = $\alpha + \beta = \frac{1}{4}$ and Product of zeroes = $\alpha\beta = -1$

If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial =

$$x^2 - (\text{Sum of zeroes})x + \text{Product of zeroes}$$

$$= x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - (1/4)x + (-1)$$

$$= x^2 - x/4 - 1$$

$$= (4x^2 - x - 4)/4$$

We can consider $4x^2 - x - 4$ as a required quadratic polynomial because it will also satisfy the given condition.

Hence, $4x^2 - x - 4$ is the quadratic polynomial.

(ii) $\sqrt{2}$, $1/3$

Given: Sum of zeroes = $\alpha + \beta = \sqrt{2}$ and Product of zeroes = $\alpha\beta = 1/3$

$\therefore$ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial = $x^2 - (\text{Sum of zeroes})x + \text{Product of zeroes}$

$$= x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - (\sqrt{2})x + (1/3)$$

$$= (3x^2 - 3\sqrt{2}x + 1)/3$$

We can consider $3x^2 - 3\sqrt{2}x + 1$ as a required quadratic polynomial because it will also satisfy the given condition.

Hence, $3x^2 - 3\sqrt{2}x + 1$ is the quadratic polynomial.

(iii) 0 , $\sqrt{5}$

Given: Sum of zeroes = $\alpha + \beta = 0$ and Product of zeroes = $\alpha\beta = \sqrt{5}$

$\therefore$ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial =

$$x^2 - (\text{Sum of zeroes})x + \text{Product of zeroes}$$

$$= x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - (0)x + \sqrt{5}$$

$$= x^2 + \sqrt{5}$$

Hence, $x^2 + \sqrt{5}$ is the quadratic polynomial.

(iv) 1, 1

Given: Sum of zeroes = $\alpha + \beta = 1$ and Product of zeroes = $\alpha\beta = 1$

$\therefore$ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial =

$$x^2 - (\text{Sum of zeroes})x + \text{Product of zeroes}$$

$$= x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - x + 1$$

Hence, $x^2 - x + 1$ is the quadratic polynomial.

(v) $-1/4, 1/4$

Given: Sum of zeroes = $\alpha + \beta = -1/4$ and Product of zeroes = $\alpha\beta = 1/4$

$\therefore$ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial =

$$x^2 - (\text{Sum of zeroes})x + \text{Product of zeroes}$$

$$= x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - (-1/4)x + (1/4)$$

$$= x^2 + x/4 + 1/4$$

$$= (4x^2 + x + 1)/4$$

We can consider $4x^2 + x + 1$ as a required quadratic polynomial because it will also satisfy the given condition.

Hence, $4x^2 + x + 1$ is the quadratic polynomial.

(vi) 4, 1

Given: Sum of zeroes = $\alpha + \beta = 4$ and Product of zeroes = $\alpha\beta = 1$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial =

$$x^2 - (\text{Sum of zeroes})x + \text{Product of zeroes}$$

$$= x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - 4x + 1$$

Hence, $x^2 - 4x + 1$ is the quadratic polynomial.