



Exercise Set 11.1 – Solutions



Chapter 11: The World of Algorithms

1. **+** Add two 4-digit numbers using the given algorithm

Let us take:

$$4738 + 6957$$

Following the algorithm precisely:

$$\begin{array}{r} 4738 \\ + 6957 \\ \hline \end{array}$$

Step 1: Align the digits from right to left.

Step 2: Add the rightmost digits:

$$8 + 7 = 15$$

Write **5** and carry **1**.

Step 3: Move to the next column:

$$3 + 5 + 1 = 9$$

Write **9** and carry **0**.

Next:

$$7 + 9 = 16$$

Write **6** and carry **1**.

Next:

$$4 + 6 + 1 = 11$$

Write **1** and carry **1**.

Step 4: There are no more digits on the left.

Step 5: Since the carry is 1, write **1** to the left.

Therefore:

$$\begin{array}{r} 4738 \\ + 6957 \\ \hline 11695 \end{array}$$

Answer: 11695

The algorithm gives the correct result when its steps are followed precisely.

2. What happens if you add a 5-digit number to a 3-digit number?

Yes, the steps handle this situation correctly.

The important point is that the numbers must be **aligned from right to left**.

For example:

$$\begin{array}{r} 12345 \\ + 678 \\ \hline \end{array}$$

Aligning from the right:

$$\begin{array}{r} 12345 \\ + 00678 \\ \hline \end{array}$$

Now add:

$$5 + 8 = 13 \rightarrow \text{write 3, carry 1}$$

$$4 + 7 + 1 = 12 \rightarrow \text{write 2, carry 1}$$

$$3 + 6 + 1 = 10 \rightarrow \text{write 0, carry 1}$$

$$2 + 0 + 1 = 3$$

$$1 + 0 = 1$$

Therefore:

$$\begin{array}{r} 12345 \\ + 678 \\ \hline 13023 \end{array}$$

Answer:

The algorithm works correctly because the columns are aligned from **right to left**, and the missing digits in the shorter number effectively contribute 0.

3. Why is it important to align the columns from right to left?

In our place-value system:

- Ones are on the right.
- Tens are to their left.
- Hundreds are further left.
- Thousands are further left.

Therefore, corresponding place values must be in the same column.

For example:

$$\begin{array}{r} 1234 \\ + 567 \end{array}$$

must be treated as:

$$\begin{array}{r} 1234 \\ + 0567 \end{array}$$

This ensures that:

- Ones are added to ones.
- Tens are added to tens.
- Hundreds are added to hundreds.
- Thousands are added to thousands.

Answer:

Columns must be aligned from **right to left** so that digits with the same place value are added together.

4. Why cannot the value of carry be more than 1?

In Step 3, we add:

two single digits + the current carry.

The largest possible digits are:

$$9 + 9 + 1 = 19$$

The largest possible sum is therefore **19**.

When the sum is 10 or more, only **1** ten is carried to the next column.

For example:

$$9 + 9 + 1 = 19$$

We write **9** and carry **1**.

Answer:

The carry cannot be more than **1** because the maximum possible sum is **19**, which contains only one group of ten.

5. What happens if we do not include the fifth step?

The fifth step says:

If the value of carry is 1, write 1 to the left of the bottom row.

If we remove this step, the final carry may be **lost**.

Example where the algorithm still works

Consider:

$$1234 + 2345$$

$$\begin{array}{r}
 1234 \\
 + 2345 \\
 \hline
 3579
 \end{array}$$

There is **no carry after the final column**, so removing Step 5 does not cause a problem.

Example where the algorithm fails

Consider:

$$5738 + 6957$$

The final calculation produces a carry:

$$\begin{array}{r}
 5738 \\
 + 6957 \\
 \hline
 12695
 \end{array}$$

If Step 5 is omitted, the final carry **1** is not written.

The result would incorrectly become:

$$2695$$

instead of:

$$12695$$

Answer:

Without Step 5:

- The algorithm works correctly when the **final carry is 0**.
- It fails when the **final carry is 1**, because that carry is not written.

Therefore, **Step 5 is necessary to obtain the correct answer in all cases.**