



Exercise Set 11.3 – Solutions



Chapter 11: The World of Algorithms

1. Finding Divisors in Reverse Order

Question:

How would our original algorithm change if we computed the divisors of n by examining the numbers from 1 to n in reverse order, from n down to 1?

Solution:

The original algorithm examines:

1, 2, 3, ..., n

and adds a number to the list if it divides n .

If we examine the numbers in reverse order, we would check:

$n, n-1, n-2, \dots, 2, 1$

For each number j :

- Check whether j divides n .
- If yes, add j to the list.
- Continue until we reach 1.

Example:

For $n = 18$, we would examine:

18, 17, 16, ..., 3, 2, 1

The divisors would be found in this order:

18, 9, 6, 3, 2, 1

So the main change is that the resulting divisor list will be in **decreasing order** instead of increasing order.

✓ **Answer:** Examine the numbers from **n down to 1** and add each number that divides **n**. The divisor list will be in decreasing order.

2. 🔍 Looking for Common Divisors Backwards

Question:

What about the last algorithm described above? What happens when we look at common divisors starting from $\min(m, n)$ and work backwards to 1?

Solution:

Instead of finding **all** divisors of both numbers and then comparing the lists, we can start with:

$\min(m, n)$

because a common divisor of **m** and **n** cannot be greater than the smaller of the two numbers.

Then check the numbers in decreasing order:

$\min(m, n), \min(m, n)-1, \dots, 2, 1$

For each number, check whether it divides **both m and n**.

The **first number** that divides both is the **GCD**.

Example: Find GCD(18, 24)

Here:

$\min(18, 24) = 18$

Check backwards:

18 → does not divide 24

17 → no

16 → no

15 → no

14 → no

13 → no

12 → divides 18? No

...

6 → divides 18 and 24? **Yes**

Therefore:

$$\mathbf{GCD(18, 24) = 6}$$

Answer:

The first common divisor encountered while working backwards from **min(m,n)** is the **GCD**.

This means we can stop as soon as we find the first common divisor; there is no need to check the remaining smaller numbers.