📘 Class 9 Maths – Ganita Manjari

📐 Chapter 12: Quadrilaterals

📝 Exercise Set 12.2 – Solutions

Q1. True or False?

(i) A parallelogram with a right angle is a rectangle.

✅ Answer: True

In a parallelogram, opposite angles are equal and adjacent angles are supplementary.

If one angle is **90°**, then all four angles are **90°**.

Therefore, the parallelogram is a **rectangle**.

(ii) A rhombus with perpendicular diagonals is a square.

❌ Answer: False

A rhombus always has **perpendicular diagonals**, but it need not be a square.

For example, a rhombus can have angles that are not 90°.

Therefore, perpendicular diagonals alone do not make a rhombus a square.

(iii) If the diagonals of a parallelogram are equal, then it is a rectangle.

✅ Answer: True

A parallelogram whose diagonals are equal is a **rectangle**.

Therefore, the statement is true.

Q2. The diagonal AC of a parallelogram ABCD bisects ∠A. Show that it also bisects ∠C and that ABCD is a rhombus.

Solution:

Given:

ABCD is a parallelogram

and **AC bisects ∠A**.

Therefore,

∠BAC = ∠CAD

Since ABCD is a parallelogram:

AB ∥ CD

Therefore, using alternate interior angles:

∠BAC = ∠ACD

Also,

AD ∥ BC

Therefore,

∠CAD = ∠BCA

Hence:

∠ACD = ∠BCA

So, **AC bisects ∠C**.

Now prove that ABCD is a rhombus

Consider $\triangle$**ABC** and $\triangle$**ADC**.

We have:

$\angle$BAC = $\angle$CAD

$\angle$BCA = $\angle$ACD

and

AC = AC (common side)

Therefore,

$\triangle$ABC ≅ $\triangle$ADC by ASA.

Hence corresponding sides are equal:

AB = AD

But in a parallelogram:

AB = CD

and

AD = BC

Therefore:

AB = BC = CD = DA

Thus all four sides are equal.

✅ Therefore, ABCD is a rhombus.

Q3. Examine the converses of true properties.

(i) If the diagonals of a quadrilateral ABCD bisect its angles, must ABCD be a rhombus?

✅ Answer: Yes

If both diagonals bisect the corresponding angles of the quadrilateral, the quadrilateral has the angle-bisector property of a rhombus.

Therefore:

ABCD must be a rhombus.

Answer: YES

(ii) If the diagonals of a quadrilateral ABCD bisect each other at right angles, must ABCD be a rhombus?

✅ Answer: Yes

If the diagonals bisect each other, then the quadrilateral is a **parallelogram**.

If the diagonals are also perpendicular, then the parallelogram is a **rhombus**.

Therefore:

ABCD is a rhombus.

Answer: YES

(iii) If the diagonals of a quadrilateral ABCD are of equal length, must ABCD be a rectangle?

❌ Answer: No

Equal diagonals alone do **not** guarantee that a quadrilateral is a rectangle.

For example, an **isosceles trapezium** can have equal diagonals without being a rectangle.

Extra condition:

If the diagonals **bisect each other**, then ABCD is a parallelogram.

A parallelogram with equal diagonals is a rectangle.

Therefore, one suitable additional condition is:

The diagonals must bisect each other.

Answer: NO

Extra condition: The diagonals bisect each other.

Q4. Let ABCD be a parallelogram with AB ≠ BC. Show that the pairwise intersection points of the four angle bisectors form the vertices of a rectangle. Why did we assume AB ≠ BC?

Solution:

Let the four angle bisectors of parallelogram ABCD intersect pairwise at the points shown in the figure.

Since ABCD is a parallelogram:

$\angle A + \angle B = 180°$

Therefore, their half-angles satisfy:

$\angle A/2 + \angle B/2 = 90°$

Thus, the angle bisectors of two adjacent angles are **perpendicular**.

Similarly, the angle bisectors at the other adjacent vertices are also perpendicular.

Therefore, the four intersection points form a quadrilateral with **right angles**.

Now consider opposite angle bisectors.

Since opposite angles of a parallelogram are equal:

$\angle A = \angle C$

Therefore, their angle bisectors are parallel.

Similarly:

$\angle B = \angle D$

so their angle bisectors are parallel.

Thus, the quadrilateral formed by the four intersection points has:

- Opposite sides parallel
- All angles equal to **$90°$**

Hence it is a:

✅ Rectangle

Why did we assume $AB \neq BC$?

If:

$AB = BC$

then the parallelogram is a **rhombus**.

In a rhombus, the angle bisectors meet in such a way that the four pairwise intersection points do not form the rectangle described in the question; some of the relevant angle bisectors coincide.

Therefore, the condition:

$AB \neq BC$

ensures that the four angle bisectors produce four distinct intersection points forming a rectangle.

✅ Final Answer:

The four pairwise intersection points of the angle bisectors form a **rectangle**, and **$AB \neq BC$** is required to ensure that the four intersection points are distinct.