📘 Class 9 Maths – Ganita Manjari

📐 Chapter 12: Quadrilaterals

📝 Exercise Set 12.3 – Solutions

Q1. In ΔABC, P, Q, R are the midpoints of AB, AC and BC respectively.

(i) Show that ΔPQR is congruent to ΔQPA and to two other triangles.

Since P, Q and R are midpoints:

AP = PB = AB/2

AQ = QC = AC/2

BR = RC = BC/2

By the Midpoint Theorem:

PQ = BC/2

QR = AB/2

PR = AC/2

Now compare the triangles.

ΔPQR and ΔQPA

- PQ = QA? Careful: these are not generally equal.

Instead, match the three sides:

PQ = BC/2

QR = AB/2 = PA

PR = AC/2 = QA

Therefore:

ΔPQR ≅ ΔQPA by SSS.

Similarly:

ΔPQR ≅ ΔPRB

because:

- PQ = RB
- QR = PB
- PR = AC/2

And:

ΔPQR ≅ ΔQRC

because:

- PQ = RC
- QR = QC
- PR = QA

Therefore, the four triangles are:

ΔPQR, ΔQPA, ΔPRB and ΔQRC

and all four are congruent.

✅ Answer:

ΔPQR ≅ ΔQPA ≅ ΔPRB ≅ ΔQRC

(ii) Suppose someone erases ΔABC, leaving only ΔPQR. Can you reconstruct ΔABC from ΔPQR?

Answer: Yes.

Since:

AB = 2QR

AC = 2PR

BC = 2PQ

we can reconstruct ΔABC by constructing a triangle whose three sides are:

2PQ, 2PR and 2QR

After constructing ΔABC, mark:

- P as midpoint of AB
- Q as midpoint of AC
- R as midpoint of BC

Thus, ΔABC can be reconstructed from ΔPQR.

✅ Answer: Yes, ΔABC can be reconstructed from ΔPQR.

Q2. In ΔABC, M and N are midpoints of AB and AC respectively. Let D be any point on BC. Show that MN bisects AD.

Solution:

Given:

- M is the midpoint of **AB**
- N is the midpoint of **AC**
- D lies on **BC**

By the **Midpoint Theorem** in ΔABC:

MN ∥ BC

Since D lies on BC:

MN ∥ BD

Now consider **ΔABD**.

M is the midpoint of AB.

Through M, draw MN parallel to BD and let it meet AD at O.

By the **Converse of the Midpoint Theorem**, O is the midpoint of AD.

Therefore:

AO = OD

Hence:

✅ **MN bisects AD.**

Q3. In quadrilateral ABCD, AB ∥ DC and AB ≠ CD. G and H are the midpoints of AC and BD respectively. Prove that GH ∥ AB. Why did we assume AB ≠ CD?

Solution:

Given:

AB ∥ DC

and:

AB ≠ CD

G is the midpoint of **AC**.

H is the midpoint of **BD**.

Consider the two diagonals **AC and BD**.

Using the midpoint theorem in the appropriate triangles, the segment joining the midpoints of the two diagonals of a trapezium is parallel to its parallel sides.

Therefore:

GH // AB

Since:

AB // DC

we also have:

GH // DC

Why did we assume AB ≠ CD?

If:

AB = CD

and also:

AB // CD

then ABCD would be a **parallelogram**.

In a parallelogram, the diagonals bisect each other.

Therefore, their midpoints **G and H would coincide**.

So GH would not be a proper line segment.

Hence, the condition:

AB ≠ CD

ensures that **G and H are distinct points**, so GH is a genuine segment parallel to AB.

✅ Answer: GH // AB, and AB ≠ CD is needed so that G and H do not coincide.

Q4. P, Q, R and S are the midpoints of AB, BC, CD and DA respectively.

(i) Show that PR and QS bisect each other.

Since P and Q are midpoints of AB and BC:

By the Midpoint Theorem:

PQ ∥ AC

Similarly, S and R are midpoints of AD and CD:

SR ∥ AC

Therefore:

PQ ∥ SR

Also, Q and R are midpoints of BC and CD:

QR ∥ BD

And P and S are midpoints of AB and AD:

PS ∥ BD

Therefore:

PQRS is a parallelogram.

The diagonals of a parallelogram bisect each other.

The diagonals of PQRS are:

PR and QS

Hence:

✅ PR and QS bisect each other.

(ii) Show that if AC = BD, then PR and QS are perpendicular. Is the converse true?

From the Midpoint Theorem:

PQ = AC/2

and:

QR = BD/2

Given:

AC = BD

Therefore:

PQ = QR

But PQRS is a parallelogram.

A parallelogram with adjacent sides equal is a **rhombus**.

Therefore, **PQRS is a rhombus**.

The diagonals of a rhombus are perpendicular.

The diagonals of PQRS are **PR and QS**.

Therefore:

PR ⊥ QS

Converse

Suppose:

PR ⊥ QS

Since PQRS is a parallelogram, a parallelogram with perpendicular diagonals is a **rhombus**.

Therefore:

PQ = QR

But:

PQ = AC/2

and:

QR = BD/2

Hence:

AC/2 = BD/2

Therefore:

AC = BD

✅ **Yes, the converse is true.**

Q5. Suppose PQRS is the Varignon parallelogram of ABCD.

(i) Copy only PQRS on another paper. Show how you will recreate a congruent copy A'B'C'D' of ABCD from PQRS.

⚠️ Important point about the textbook question

Taken **literally**, if you are given **only PQRS**, a congruent copy of an arbitrary original quadrilateral ABCD cannot be uniquely reconstructed.

The reason is that many different quadrilaterals can have the **same Varignon parallelogram**. The four midpoint conditions are:

P = midpoint of A'B'

Q = midpoint of B'C'

R = midpoint of C'D'

S = midpoint of D'A'

For example, once A' is chosen, the other vertices can be constructed by reflection:

- B' is the reflection of A' in P.

- D' is the reflection of A' in S.
- C' can then be obtained using Q or R.

Different choices of A' can produce different quadrilaterals having the same PQRS.

Therefore, **PQRS alone does not contain enough information to guarantee a quadrilateral congruent to the original ABCD**.

So I would **not write a fabricated construction** for Q5(i).

(ii) There are multiple ways to construct A'B'C'D' in (i). Justify why it is congruent to ABCD.

Because of the issue above, this part also cannot be justified from **PQRS alone** for a completely arbitrary ABCD.

The textbook itself gives the hint:

$\Delta SDR \cong \Delta SD'R$

and similarly asks you to establish the corresponding collinearities.

However, to prove congruence using that route, additional information about the original quadrilateral (such as the relevant lengths/position of A or D) is needed.

⚠️ Therefore:

Q5(i)–(ii), as literally worded with only PQRS available, is underdetermined.

Q5 (iii). Show that if PQRS is a square, then AC and BD are perpendicular and equal. Prove the converse.

Part 1: If PQRS is a square

Since PQRS is the Varignon parallelogram:

PQ ∥ AC

and:

QR ∥ BD

Also, because PQRS is a square:

PQ = QR

and:

PQ ⊥ QR

Therefore, the corresponding original diagonals satisfy:

AC = BD

because:

PQ = AC/2

and:

QR = BD/2

Also:

AC ⊥ BD

because:

PQ ⊥ QR

Hence:

✅ AC and BD are perpendicular and equal.

Converse

Suppose:

AC = BD

and:

AC ⊥ BD

Since:

PQ = AC/2

and:

QR = BD/2

we get:

PQ = QR

Also:

PQ // AC

and:

QR // BD

Therefore:

PQ ⊥ QR

Thus PQRS is a parallelogram having:

- Two adjacent sides equal
- A right angle

Therefore:

PQRS is a square.

✅ Hence the converse is true.