📘 Class 9 Maths – Ganita Manjari Part II

Chapter 13: Two Variables, One Line

Exercise Set 13.5 – Complete Questions with Solutions

Q1. Form a pair of linear equations for each of the following problems and find their solutions.

(i) The sum of two integers is +5 and their difference is −21. Find the two numbers.

Let the two integers be **x and y**.

According to the question:

$x + y = 5$ …(1)

$x - y = -21$ …(2)

Adding (1) and (2):

$2x = -16$

$x = -8$

Substitute $x = -8$ in (1):

$-8 + y = 5$

$y = 13$

Answer:

The two integers are −8 and 13.

(ii) The difference between two numbers is 26 and one number is three times the other. Find the numbers.

Let the numbers be **x and y**.

Difference:

$x - y = 26$ …(1)

One number is three times the other:

$x = 3y$ …(2)

Substitute (2) in (1):

$3y - y = 26$

$2y = 26$

$y = 13$

Therefore:

$x = 3(13)$

$x = 39$

Answer:

The numbers are 39 and 13.

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹8880. Later, she buys 5 bats and 5 balls for ₹4000. Find the cost of each bat and each ball.

Let:

Cost of one bat = ₹x

Cost of one ball = ₹y

From the first purchase:

$7x + 6y = 8880$ …(1)

From the second purchase:

$5x + 5y = 4000$

Dividing by 5:

$x + y = 800$ …(2)

From (2):

$x = 800 - y$

Substitute in (1):

$7(800 - y) + 6y = 8880$

$5600 - 7y + 6y = 8880$

$5600 - y = 8880$

$-y = 3280$

This gives $y = -3280$, which is impossible for a cost.

Therefore, the given numerical data as shown in the question are **inconsistent**.

Answer:

The two equations

$7x + 6y = 8880$

$5x + 5y = 4000$

do **not give positive costs** for a bat and a ball. Hence, there appears to be an error in the printed numerical data/question.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered for every km. For a distance of 10 km, the total amount paid

is ₹155 and for a journey of 15 km, the total amount paid is ₹220. What is the fixed charge and the charge per km?

Let:

Fixed charge = ₹x

Charge per km = ₹y

For 10 km:

x + 10y = 155 …(1)

For 15 km:

x + 15y = 220 …(2)

Subtract (1) from (2):

5y = 65

y = 13

Substitute in (1):

x + 10(13) = 155

x + 130 = 155

x = 25

For 25 km:

Total fare = x + 25y

= 25 + 25(13)

= 25 + 325

= **₹350**

Answer:

- **Fixed charge = ₹25**
- **Charge per km = ₹13**
- **Fare for 25 km = ₹350**

(v) A fraction becomes equal to 9/11 if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator, it becomes equal to 5/6. Find the fraction.

Let the fraction be:

x/y

According to the question:

$(x + 2)/(y + 2) = 9/11$

$11(x + 2) = 9(y + 2)$

$11x + 22 = 9y + 18$

$11x - 9y = -4$ …(1)

Also:

$(x + 3)/(y + 3) = 5/6$

$6(x + 3) = 5(y + 3)$

$6x + 18 = 5y + 15$

$6x - 5y = -3$ …(2)

Multiply (1) by 5:

$55x - 45y = -20$

Multiply (2) by 9:

$54x - 45y = -27$

Subtract:

$x = 7$

Substitute in (2):

$6(7) - 5y = -3$

$42 - 5y = -3$

$5y = 45$

$y = 9$

Answer:

The fraction is 7/9.

(vi) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes equal to 1/2 if we add 1 only to the denominator. What is the fraction?

Let the fraction be:

x/y

First condition:

$(x + 1)/(y - 1) = 1$

Therefore:

$x + 1 = y - 1$

$y = x + 2$ …(1)

Second condition:

$x/(y + 1) = 1/2$

$2x = y + 1$ …(2)

Substitute (1) in (2):

$2x = x + 2 + 1$

$2x = x + 3$

$x = 3$

Therefore:

$y = 3 + 2 = 5$

Answer:

The fraction is 3/5.

(vii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

Let:

Present age of Nuri = x years

Present age of Sonu = y years

Five years ago:

$\mathbf{x - 5 = 3(y - 5)}$

$x - 5 = 3y - 15$

$\mathbf{x - 3y = -10}$ …(1)

Ten years later:

$\mathbf{x + 10 = 2(y + 10)}$

$x + 10 = 2y + 20$

$\mathbf{x - 2y = 10}$ …(2)

Subtract (1) from (2):

$y = 20$

Substitute in (2):

$x - 40 = 10$

x = 50

Answer:

- **Nuri = 50 years**
- **Sonu = 20 years**

(viii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Let the tens digit be **x** and the units digit be **y**.

Sum of digits:

x + y = 9 …(1)

The number is:

10x + y

The reversed number is:

10y + x

According to the question:

9(10x + y) = 2(10y + x)

90x + 9y = 20y + 2x

88x = 11y

y = 8x …(2)

Substitute in (1):

x + 8x = 9

9x = 9

x = 1

Therefore:

y = 8

The number is:

18

Verification:

9 × 18 = 162

Reversed number = 81

2 × 81 = 162

Answer:

The number is 18.

(ix) Meena went to a bank to withdraw ₹2000. She asked the cashier to give her ₹50 and ₹100 notes only. Meena got 25 notes in all. Find how many notes of ₹50 and ₹100 did she receive?

Let:

Number of ₹50 notes = x

Number of ₹100 notes = y

Total number of notes:

x + y = 25 …(1)

Total amount:

50x + 100y = 2000

Divide by 50:

x + 2y = 40 …(2)

Subtract (1) from (2):

y = 15

Substitute in (1):

x + 15 = 25

x = 10

Answer:

- **₹50 notes = 10**
- **₹100 notes = 15**

(x) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹27 for a book kept for seven days, while Susy paid ₹21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Let:

Fixed charge = ₹x

Charge for each extra day = ₹y

For 7 days, there are 4 extra days:

x + 4y = 27 …(1)

For 5 days, there are 2 extra days:

x + 2y = 21 …(2)

Subtract (2) from (1):

2y = 6

y = 3

Substitute in (2):

$x + 2(3) = 21$

$x = 15$

Answer:

- **Fixed charge = ₹15**
- **Charge for each extra day = ₹3**

Q2. Form a pair of linear equations and find their common solutions graphically. 10 students of Grade 9 took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Let:

Number of boys = x

Number of girls = y

Total students:

$x + y = 10$ …(1)

Girls are 4 more than boys:

$y = x + 4$

or

$-x + y = 4$ …(2)

Solving:

$x + y = 10$

$-x + y = 4$

Adding:

$2y = 14$

$y = 7$

Therefore:

$x = 3$

Answer:

- **Boys = 3**
- **Girls = 7**

Graphically, the two lines intersect at:

(3, 7)

Q3. Use the ratios a_1/a_2, b_1/b_2, c_1/c_2 to determine whether the lines representing the following pairs of linear equations intersect at a point, are parallel or are coincident.

(i) $5x - 4y + 8 = 0$; $7x + 6y - 9 = 0$

Here:

$a_1 = 5$, $b_1 = -4$, $c_1 = 8$

$a_2 = 7$, $b_2 = 6$, $c_2 = -9$

Compare:

$a_1/a_2 = 5/7$

$b_1/b_2 = -4/6 = -2/3$

Since:

$5/7 \neq -2/3$

the lines intersect at one point.

Answer:

Intersecting lines

(ii) $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$

$a_1/a_2 = 9/18 = 1/2$

$b_1/b_2 = 3/6 = 1/2$

$c_1/c_2 = 12/24 = 1/2$

Therefore:

$a_1/a_2 = b_1/b_2 = c_1/c_2$

Answer:

Coincident lines

(iii) $6x - 3y + 10 = 0$; $2x - y + 9 = 0$

$a_1/a_2 = 6/2 = 3$

$b_1/b_2 = -3/-1 = 3$

$c_1/c_2 = 10/9$

Therefore:

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Answer:

Parallel lines

Q4. Which of the following pairs of linear equations have solutions? If they have solutions, find them graphically.

(i) $x + y = 5$, $2x + 2y = 10$

The second equation divided by 2 gives:

$x + y = 5$

Both equations represent the same line.

Therefore, they have **infinitely many solutions**.

Answer:

Infinitely many solutions.

(ii) $x - y = 8$, $3x - 3y = 16$

Divide the second equation by 3:

$x - y = 16/3$

The two lines have the same coefficients but different constants.

Therefore, they are parallel.

Answer:

No solution.

(iii) $2x + y - 6 = 0, 4x - 2y - 4 = 0$

Rewrite:

$2x + y = 6$ …(1)

$4x - 2y = 4$

Divide by 2:

$2x - y = 2$ …(2)

Add (1) and (2):

$4x = 8$

$x = 2$

Substitute in (1):

$2(2) + y = 6$

$y = 2$

Answer:

$x = 2, y = 2$

The lines intersect at **(2, 2)**.

(iv) $2x - 2y - 2 = 0, 4x - 4y - 5 = 0$

Rewrite:

$2x - 2y = 2$

$4x - 4y = 5$

Compare:

$a_1/a_2 = 2/4 = 1/2$

$b_1/b_2 = -2/-4 = 1/2$

but

$c_1/c_2 = 2/5$

Therefore:

$\mathbf{a_1/a_2 = b_1/b_2 \neq c_1/c_2}$

The lines are parallel.

Answer:

No solution.

Q5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Let:

Length = x m

Width = y m

Half the perimeter:

$\mathbf{x + y = 36}$ …(1)

Length is 4 m more than width:

$\mathbf{x - y = 4}$ …(2)

Add (1) and (2):

$2x = 40$

$x = 20$

Substitute:

$20 + y = 36$

$y = 16$

Answer:

Length = 20 m

Width = 16 m

Q6. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables so that the graphs of the pair formed represent:

(i) Intersecting lines

We need an equation with a different slope.

For example:

$\mathbf{x + y - 3 = 0}$

So the pair is:

$\mathbf{2x + 3y - 8 = 0}$

$\mathbf{x + y - 3 = 0}$

These represent intersecting lines.

Answer:

$\mathbf{x + y - 3 = 0}$

(ii) Parallel lines

We need the same ratio of x and y coefficients but a different constant.

Multiply the given equation by 2:

$4x + 6y - 16 = 0$

Now change the constant:

$4x + 6y - 10 = 0$

Therefore:

$2x + 3y - 8 = 0$

$4x + 6y - 10 = 0$

represent parallel lines.

Answer:

$4x + 6y - 10 = 0$

(iii) Coincident lines

Take a non-zero multiple of the given equation:

$4x + 6y - 16 = 0$

Both equations represent the same line.

Answer:

$4x + 6y - 16 = 0$

Q7. Here is a problem that was posed by Mahāvīrācārya in Ganita sāra saṅgraha (c. 850 CE).

The price of 9 citrons and 7 fragrant wood-apples taken together is 107; and the price of 7 citrons and 9 fragrant wood-apples taken together is 101. O mathematician, tell me quickly the price of each citron and of each fragrant wood-apple.

Let:

Price of one citron = x

Price of one fragrant wood-apple = y

Then:

$\mathbf{9x + 7y = 107}$ …(1)

$\mathbf{7x + 9y = 101}$ …(2)

Add the two equations:

$16x + 16y = 208$

Divide by 16:

$\mathbf{x + y = 13}$ …(3)

Now subtract (2) from (1):

$2x - 2y = 6$

Divide by 2:

$\mathbf{x - y = 3}$ …(4)

Add (3) and (4):

$2x = 16$

$x = 8$

Substitute in (3):

$8 + y = 13$

$y = 5$

Answer:

- **Price of one citron = 8**

- **Price of one fragrant wood-apple = 5**

Q8. 5 pencils and 7 pens together cost ₹50, whereas 7 pencils and 5 pens together cost ₹46. Find the cost of each pencil and pen.

Let:

Cost of one pencil = ₹x

Cost of one pen = ₹y

From the first statement:

$5x + 7y = 50$ …(1)

From the second:

$7x + 5y = 46$ …(2)

Add:

$12x + 12y = 96$

$x + y = 8$ …(3)

Subtract (1) from (2):

$2x − 2y = −4$

$x − y = −2$ …(4)

Add (3) and (4):

$2x = 6$

$x = 3$

Substitute in (3):

3 + y = 8

y = 5

Answer:

- **Cost of one pencil = ₹3**
- **Cost of one pen = ₹5**

Q9. Find the height of the stool.

From the figure:

When the cat is sitting on the stool:

Height of stool + height of cat = 85 cm

When the cat is standing beside the stool:

Height of stool − height of cat = 25 cm

Let:

Height of stool = x cm

Height of cat = y cm

Therefore:

x + y = 85 …(1)

x − y = 25 …(2)

Add:

2x = 110

x = 55

Answer:

The height of the stool is 55 cm.