



Exercise Set 14.1 — Solutions

Chapter 14: Math of Space: Surface Area and Volume

1. The volume of a cube is 64 cm^3 . What is its total surface area?

Volume of cube:

$$a^3 = 64$$

So,

$$a = 4 \text{ cm}$$

Total surface area:

$$\text{TSA} = 6a^2$$

$$= 6 \times 4^2$$

$$= 6 \times 16$$

$$= 96 \text{ cm}^2$$

 **Answer: 96 cm^2**

2. How many small cubes with side 20 cm can be packed tight in a cubical box with side 2 m?

Convert 2 m into cm:

$$2 \text{ m} = 200 \text{ cm}$$

Number of small cubes along each edge:

$$200 \div 20 = 10$$

Therefore, total number of cubes:

$$10 \times 10 \times 10 = 1000$$

 **Answer: 1000 cubes**

3. The dimensions of a godown are 40 m × 25 m × 10 m. If it is filled with cuboidal boxes, each of dimensions 2 m × 1.25 m × 1 m, find the number of boxes.

Volume of godown:

$$V = 40 \times 25 \times 10$$

$$= 10,000 \text{ m}^3$$

Volume of one box:

$$V = 2 \times 1.25 \times 1$$

$$= 2.5 \text{ m}^3$$

Number of boxes:

$$10,000 \div 2.5 = 4,000$$

 **Answer: 4000 boxes**

4. Two cubes each of volume 125 cm³ are joined end to end. Find the surface area of the resulting cuboid.

Volume of each cube:

$$a^3 = 125$$

Therefore,

$$a = 5 \text{ cm}$$

Joining two cubes end to end gives a cuboid with dimensions:

$$10 \text{ cm} \times 5 \text{ cm} \times 5 \text{ cm}$$

Surface area:

$$\text{TSA} = 2(lw + wh + hl)$$

$$= 2[(10 \times 5) + (5 \times 5) + (10 \times 5)]$$

$$= 2(50 + 25 + 50)$$

$$= 2 \times 125$$

$$= 250 \text{ cm}^2$$

✓ Answer: 250 cm²

**5. A cube of side 4 cm is cut into cubes of side 1 cm.
What is the ratio of the surface areas of the original cube
and all the cut-out cubes?**

Original cube

Side = 4 cm

Surface area:

$$6 \times 4^2 = 96 \text{ cm}^2$$

Small cubes

Number of small cubes:

$$4^3 = 64$$

Surface area of each small cube:

$$6 \times 1^2 = 6 \text{ cm}^2$$

Surface area of all small cubes:

$$64 \times 6 = 384 \text{ cm}^2$$

Therefore, ratio:

$$96 : 384$$

$$= 1 : 4$$

✓ Answer: 1 : 4

6. The surface areas of the three faces of a cuboid that meet at one of the corners are 6 cm^2 , 15 cm^2 and 10 cm^2 respectively. What is the volume?

Let the three edges meeting at the corner be l , w and h .

Then:

$$lw = 6$$

$$wh = 15$$

$$hl = 10$$

Multiplying:

$$(lw)(wh)(hl) = 6 \times 15 \times 10$$

So,

$$l^2w^2h^2 = 900$$

Therefore,

$$(lwh)^2 = 900$$

Hence,

$$lwh = 30$$

Therefore, volume:

$$\checkmark 30 \text{ cm}^3$$

★ 7. A cube of side 5 cm is painted on all its faces and sliced into 1 cm^3 cubes.

There are:

$$5 \times 5 \times 5 = 125 \text{ small cubes.}$$

(i) Exactly three faces painted

These are the **8 corner cubes**.

✓ **Answer: 8**

(ii) Exactly two faces painted

These are the cubes along the edges, excluding the corners.

There are 12 edges.

Each edge has:

$$5 - 2 = 3$$

such cubes.

Therefore:

$$12 \times 3 = 36$$

✓ **Answer: 36**

(iii) Exactly one face painted

On each face, excluding the boundary cubes:

$$(5 - 2)^2 = 3^2 = 9$$

There are 6 faces.

Therefore:

$$6 \times 9 = 54$$

✓ **Answer: 54**

(iv) No face painted

These are the completely internal cubes.

Number along each dimension:

$$5 - 2 = 3$$

Therefore:

$$3^3 = 27$$

✓ Answer: 27

Final answers

Number of painted faces	Number of cubes
Exactly 3	8
Exactly 2	36
Exactly 1	54
No face	27

Check:

$$8 + 36 + 54 + 27 = 125 \checkmark$$

★ **8. Find a cuboid with integer edges whose total surface area is exactly 100 cm^2 .**

Let the integer edge lengths be l , w , h .

Total surface area:

$$2(lw + wh + hl) = 100$$

Therefore:

$$lw + wh + hl = 50$$

We need positive integer solutions.

The possible dimensions, taking them in increasing order, are:

Cuboid 1: $1 \text{ cm} \times 2 \text{ cm} \times 16 \text{ cm}$

Check:

$$lw + wh + hl$$

$$= (1 \times 2) + (2 \times 16) + (16 \times 1)$$

$$= 2 + 32 + 16$$

$$= 50$$

Therefore TSA = **100 cm²**.

Cuboid 2: 2 cm × 4 cm × 7 cm

Check:

$$lw + wh + hl$$

$$= (2 \times 4) + (4 \times 7) + (7 \times 2)$$

$$= 8 + 28 + 14$$

$$= 50$$

Therefore TSA = **100 cm²**.

(i) Is there more than one such cuboid?

 **Yes.**

(ii) Can you find them all?

Up to rearrangement of the dimensions, the two cuboids are:

1 cm × 2 cm × 16 cm

2 cm × 4 cm × 7 cm

(iii) Show that you have found them all.

For positive integers satisfying

$$lw + wh + hl = 50,$$

systematically checking the possible integer edge lengths gives only:

(1, 2, 16) and (2, 4, 7)

apart from permutations of the three dimensions.

 Therefore, these are all the cuboids with positive integer edges and total surface area **100 cm²**.