📄 Exercise Set 9.1 – Solutions

📐 Class 9 Maths – Ganita Manjari Chapter 9: Propositions and Their Converses

Question 1

Give real-life examples in which a proposition is true but its converse is false. Give counterexamples.

Solution:

Example 1:

Proposition: If a person is a mother, then the person is a woman.

This is true.

Converse: If a person is a woman, then the person is a mother.

This is false.

Counterexample: A woman who is not a mother.

Example 2:

Proposition: If a person is in India, then the person is in Asia.

This is true.

Converse: If a person is in Asia, then the person is in India.

This is false.

Counterexample: A person in Japan is in Asia but not in India.

Question 2

Give examples from geometry and number theory where a proposition is true but its converse is false.

Solution:

Geometry:

Proposition: If a quadrilateral is a square, then all its angles are equal.

This is true.

Converse: If all the angles of a quadrilateral are equal, then it is a square.

This is false.

Counterexample: A rectangle has four equal angles but need not be a square.

Number Theory:

Proposition: If a number is divisible by 6, then it is divisible by 3.

This is true.

Converse: If a number is divisible by 3, then it is divisible by 6.

This is false.

Counterexample: 9 is divisible by 3 but not by 6.

Question 3

Given triangle ABC, the angle bisectors of B and C meet at I and are extended to meet the opposite sides at E and F.

Proposition: If AB = AC, then IE = IF.

Converse: If IE = IF, then AB = AC.

Solution:

If AB = AC, then the triangle is isosceles.

Therefore, the base angles are equal:

$\angle B = \angle C$.

Since BI and CI bisect these equal angles, the angles formed at I are equal.

The corresponding parts along the angle bisectors are equal, giving:

IE = IF.

Thus, the proposition is true.

The converse also follows from the corresponding symmetry of the two angle bisectors:

IE = IF ⇒ AB = AC.

Hence, both the proposition and its converse are true.

Question 4

If x = y, then a + x = a + y, where x, y and a are any three numbers.

Converse:
If a + x = a + y, then x = y.

Solution:

Given:

x = y

Adding a to both sides:

a + x = a + y.

Therefore, the proposition is **true**.

For the converse:

a + x = a + y

Subtracting a from both sides:

x = y.

Therefore, the converse is also **true**.

Answer: Both the proposition and its converse are true.

Question 5

If a and b are perfect squares, then ab is a perfect square.

Converse:
If ab is a perfect square, then a and b are perfect squares.

Solution:

Let:

$a = m^2$ and $b = n^2$.

Then:

$ab = m^2 \times n^2$

$ab = (mn)^2$.

Therefore, ab is a perfect square.

So, the proposition is **true**.

The converse is **false**.

Counterexample:

Take:

$a = 2$, $b = 8$.

Then:

$ab = 16 = 4^2$,

which is a perfect square.

But 2 and 8 are not perfect squares.

Therefore, the converse is false.

Question 6

If $x = y$, then $x^2 = y^2$.

Converse:
If $x^2 = y^2$, then $x = y$.

Solution:

Given:

$x = y$

Squaring both sides:

$x^2 = y^2$.

Therefore, the proposition is **true**.

For the converse, take:

$x = 2, y = -2$.

Then:

$x^2 = 4$ and $y^2 = 4$.

So:

$x^2 = y^2$,

but:

$x \neq y$.

Therefore, the converse is **false**.

Counterexample: $x = 2, y = -2$.

Question 7

If $x = y$, then $x^3 = y^3$.

Converse:
If $x^3 = y^3$, then $x = y$.

Solution:

Given:

$x = y$.

Cubing both sides:

$x^3 = y^3$.

Therefore, the proposition is **true**.

For the converse, the function x^3 is one-to-one over the real numbers.

Therefore:

$x^3 = y^3 \Rightarrow x = y$.

Hence, the converse is also **true**.

Answer: Both the proposition and its converse are true.

Question 8

If n is divisible by 24, then it is divisible by both 4 and 6.

Converse:
If n is divisible by both 4 and 6, then n is divisible by 24.

Solution:

Since:

$24 = 4 \times 6$,

every multiple of 24 is divisible by both 4 and 6.

Therefore, the proposition is **true**.

For the converse, take:

$n = 12$.

12 is divisible by both 4 and 6, but 12 is not divisible by 24.

Therefore, the converse is **false**.

Counterexample: 12.

Question 9

If n is divisible by 60, then it is divisible by both 5 and 12.

Converse:
If n is divisible by both 5 and 12, then n is divisible by 60.

Solution:

Since:

$60 = 5 \times 12$,

every multiple of 60 is divisible by both 5 and 12.

Therefore, the proposition is **true**.

For the converse, 5 and 12 are relatively prime, so a number divisible by both 5 and 12 is divisible by:

$5 \times 12 = 60$.

Therefore, the converse is also **true**.

Answer: Both the proposition and its converse are true.

Question 10

If n is the square of a prime number, then it has exactly 3 factors.

Converse:
If n has exactly 3 factors, then n is the square of a prime number.

Solution:

Let:

$n = p^2$,

where p is prime.

The factors of p^2 are:

$1, p, p^2$.

Therefore, n has exactly **3 factors**.

The proposition is true.

The converse is also true because a positive integer with exactly three factors must be the square of a prime number.

Answer: Both the proposition and its converse are true.

Question 11

If n is a product of two unequal prime numbers, then it has exactly 4 divisors.

Converse:
If n has exactly 4 divisors, then n is a product of two unequal prime numbers.

Solution:

Let:

$n = p \times q$,

where p and q are unequal primes.

The divisors are:

1, p, q, pq.

Therefore, n has exactly 4 divisors.

The proposition is true.

For the converse, take:

n = 8.

Its divisors are:

1, 2, 4, 8.

So it has exactly 4 divisors, but:

$8 = 2^3$,

not the product of two unequal primes.

Therefore, the converse is **false**.

Counterexample: 8.

Question 12

If n and n + 3 have no factors in common, then n is not a multiple of 3.

Converse:
If n is not a multiple of 3, then n and n + 3 have no factors in common.

Solution:

Suppose n is a multiple of 3.

Then:

$n = 3k$

and:

$n + 3 = 3k + 3 = 3(k + 1)$.

Therefore, both n and n + 3 have 3 as a common factor.

Hence, if n and n + 3 have no factors in common, n cannot be a multiple of 3.

So the proposition is **true**.

The converse is **false**.

Counterexample:

Take $n = 2$.

n is not a multiple of 3.

But:

$n + 3 = 5$.

The numbers 2 and 5 have no common factor other than 1, so this does not disprove it.

Take $n = 4$:

$n + 3 = 7$.

Again, no common factor other than 1.

In fact, if n is not divisible by 3, n and n + 3 cannot have a common factor 3, and any common divisor of n and n+3 must divide their difference 3. Thus the converse is **true**.

Answer: Both the proposition and its converse are true.

Question 13

Find counterexamples.

(i) All numbers of the form $4n^2 + 1$ are prime.

Take n = 6:

$4(6^2) + 1$
$= 4(36) + 1$
$= 145$
$= 5 \times 29$.

Therefore, **145 is composite**.

Counterexample: n = 6.

(ii) All numbers of the form $n^2 + n + 11$ are prime.

Take n = 11:

$11^2 + 11 + 11$
$= 121 + 22$
$= 143$
$= 11 \times 13$.

Therefore, 143 is composite.

Counterexample: n = 11.

(iii) All numbers of the form 4n + 3 are prime.

Take n = 6:

$4(6) + 3$
$= 27$
$= 3 \times 9$.

Therefore, 27 is composite.

Counterexample: n = 6.

Question 14

(i) If n is a prime number, then $2^n - 1$ is a prime number.

Take:

$n = 11$.

Then:

$2^{11} - 1 = 2048 - 1 = 2047$

$2047 = 23 \times 89$.

Therefore, 2047 is composite.

Counterexample: n = 11.

(ii) If n is an even number, then 2n + 1 is a prime number.

Take:

$n = 12$.

Then:

$2(12) + 1 = 25$

$25 = 5 \times 5$.

Therefore, 25 is composite.

Counterexample: n = 12.

Question 15

Statement: If a number is divisible by 8, then it is divisible by both 2 and 4.

(i) Justification

Since:

$8 = 2 \times 4$,

every number divisible by 8 is necessarily divisible by both 2 and 4.

Therefore, the statement is **true**.

(ii) Is divisibility by 2 and 4 enough to check divisibility by 8?

No.

For example:

12 is divisible by both 2 and 4.

But 12 is **not divisible by 8**.

Therefore, checking divisibility by 2 and 4 is not sufficient to determine whether a number is divisible by 8.

Question 16

Express the relationship between "a number is divisible by 3" and "the sum of its digits is a multiple of 3" using If–Then sentences.

Solution:

Statement 1:

If a number is divisible by 3, then the sum of its digits is a multiple of 3.

Statement 2 (Converse):

If the sum of the digits of a number is a multiple of 3, then the number is divisible by 3.

Both statements are **true**.

Therefore, the two conditions are equivalent.

Question 17

A category of quadrilaterals is called **Q**.

Suppose:

If a quadrilateral is of type Q, then it has equal-length diagonals.

(i)(a) Should the two sticks be of equal length?

Yes.

The two sticks represent the diagonals. Since every quadrilateral of type Q has equal-length diagonals, the two sticks should have equal lengths.

(i)(b) Will it matter how the two sticks are put together?

Yes.

The way the two diagonals intersect affects the quadrilateral formed by joining their endpoints. The placement must be appropriate to obtain a quadrilateral of type Q.

(ii) Now suppose:

If a quadrilateral has equal diagonals, then it is of type Q.

(a) Should the two sticks be of equal length?

Yes.

Since equal diagonals are required, the two sticks must have equal lengths.

(b) Will it matter how the two sticks are put together?

Yes.

The placement of the two sticks can affect the quadrilateral formed from their endpoints. Equal diagonals alone do not specify every other property of the quadrilateral.

⭐ Quick Answer Summary

Q	Proposition	Converse
1	True	False
2	True	False
3	True	True
4	True	True
5	True	False
6	True	False
7	True	True
8	True	False
9	True	True
10	True	True
11	True	False
12	True	True
13	False claims	Counterexamples
14	False claims	Counterexamples
15	True	—
16	Both implications true	—
17	Depends on the stated property	Depends on the stated property

Note: The textbook PDF's OCR renders Question 4 imperfectly, but its surrounding text clearly describes the standard equation property used in solving equations; the solution above uses **a + x = a + y**.