

Polynomials NCERT Exemplar PYQs Class 10 MATH

EX 2.1

1. If one of the zeroes of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is -3, then the value of k is

(A) $\frac{4}{3}$

(B) $\frac{-4}{3}$

(C) $\frac{2}{3}$

(D) $\frac{-2}{3}$

[CBSE 2018,15,13,10,09]

Answer Option (A) $\frac{4}{3}$ is correct

Solution:

Let $p(x) = (k - 1)x^2 + kx + 1$

Given that, one of the zeroes of the quadratic polynomial $p(x) = (k - 1)x^2 + kx + 1$ is -3, then $p(-3) = 0$

$$\Rightarrow (k - 1)(-3)^2 + k(-3) + 1 = 0$$

$$\Rightarrow 9(k - 1) - 3k + 1 = 0$$

$$\Rightarrow 9k - 9 - 3k + 1 = 0$$

$$\Rightarrow 6k - 8 = 0$$

$$\Rightarrow 6k = 8$$

$$\Rightarrow k = \frac{8}{6}$$

$$\Rightarrow k = \frac{4}{3}$$

Hence the value of k is $\frac{4}{3}$.

2. A quadratic polynomial, whose zeroes are -3 and 4, is

(A) $x^2 - x + 12$

(B) $x^2 + x + 12$

(C) $\frac{x^2}{2} - \frac{x}{2} - 6$

(D) $2x^2 + 2x - 24$

[CBSE 2020,15,13,12]

Answer Option (C) $x^2/2 - x/2 - 6$ is correct

Solution:

Let the zeroes of a quadratic polynomial be $\alpha = -3$ and $\beta = 4$.

Then, the sum of zeroes = $\alpha + \beta = (-3 + 4) = 1$

and the product of zeroes = $\alpha\beta = (-3 \times 4) = -12$

Required polynomial $p(x) = k\{x^2 - (\text{sum of zeroes})x + (\text{product of zeroes})\}$

$$= k\{x^2 - (1)x + (-12)\}$$

$$= x^2 - x - 12$$

Taking $k = \frac{1}{2}$

$$= x^2/2 - x/2 - 12/2$$

$$= x^2/2 - x/2 - 6$$

Hence, the quadratic polynomial whose zeroes are -3 and 4 is $x^2/2 - x/2 - 6$.

3. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and -3, then

(A) $a = -7, b = -1$ (B) $a = 5, b = -1$ (C) $a = 2, b = -6$ (D) $a = 0, b = -6$

[CBSE 2016]

Answer: Option (D) $a = 0, b = -6$ is correct.

Solution:

$$\text{Let } p(x) = x^2 + (a + 1)x + b$$

Given that 2 and -3 are the zeroes of the given quadratic polynomial $p(x)$.

$$\therefore p(2) = 0 \text{ and } p(-3) = 0$$

$$p(2) = 0$$

$$\Rightarrow 2^2 + (a + 1)(2) + b = 0$$

$$\Rightarrow 4 + 2a + 2 + b = 0$$

$$\Rightarrow 2a + b + 6 = 0 \dots\dots (1)$$

$$\text{Now, } p(-3) = 0$$

$$\Rightarrow (-3)^2 + (a + 1)(-3) + b = 0$$

$$\Rightarrow 9 - 3a - 3 + b = 0$$

$$\Rightarrow -3a + b + 6 = 0 \dots\dots(2)$$

Solving (1) and (2), we get

$$2a + b + 6 = -3a + b + 6$$

$$2a + 3a + b - b = 6 - 6$$

$$5a = 0$$

$$a = 0$$

Putting the value of a in eq 1:

$$2a + b + 6 = 0$$

$$2(0) + b + 6 = 0$$

$$0 + b + 6 = 0$$

$$b = -6$$

Hence, if the zeros of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and -3, then the required values of a and b are $a = 0$ and $b = -6$.

5. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is

(A) $-\frac{c}{a}$

(B) $\frac{c}{a}$

(C) 0

(D) $-\frac{b}{a}$

[CBSE 2012]

Answer: Option (B) c/a is correct

Solution:

It is given that one of the zeroes of the cubic polynomial $p(x)$ is zero.

$$\text{Let } p(x) = ax^3 + bx^2 + cx + d$$

Let α, β , and γ be the zeroes of $p(x)$, where $\alpha = 0$.

We know that:

Sum of the product of zeroes taken two at a time = coefficient of $x \div$ coefficient of x^3 i.e.,

$$\text{Sum of the product of zeroes taken two at a time} = c/a$$

$$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = c/a$$

$$\Rightarrow 0 \times \beta + \beta\gamma + \gamma \times 0 = c/a$$

$$[\alpha = 0]$$

$$\Rightarrow 0 + \beta\gamma + 0 = c/a$$

$$\Rightarrow \beta\gamma = c/a$$

Hence, the product of other two zeroes is c/a .

6. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1 , then the product of the other two zeroes is

(A) $b - a + 1$ (B) $b - a - 1$ (C) $a - b + 1$ (D) $a - b - 1$ [CBSE 2012]

Answer Option (A) $b - a + 1$ is correct

Solution:

$$\text{Let } p(x) = x^3 + ax^2 + bx + c$$

Let α, β and γ be the zeroes of $p(x) = x^3 + ax^2 + bx + c$

where $\alpha = -1$

$$\text{Now, } p(-1) = 0$$

$$\Rightarrow (-1)^3 + a(-1)^2 + b(-1) + c = 0$$

$$\Rightarrow -1 + a - b + c = 0$$

$$\Rightarrow c = 1 - a + b \dots\dots\dots(1)$$

We know that product of the zeroes = $-\text{constant term} \div \text{coefficient of } x^3 = -c/1$

$$\Rightarrow \alpha\beta\gamma = -c$$

$$\Rightarrow (-1)\beta\gamma = -c$$

$$[\alpha = -1]$$

$$\Rightarrow \beta\gamma = c$$

$$\Rightarrow \beta\gamma = 1 - a + b \text{ [from eq .1]}$$

Hence, the product of the other two zeroes is $1 - a + b$ or $b - a + 1$.

EX 2.3

Find the zeroes of the following polynomials by factorisation method and verify the relations between the zeroes and the coefficients of the polynomials:

1. $4x^2 - 3x - 1$ 2. $3x^2 + 4x - 4$ 3. $5t^2 + 12t + 7$ 4. $t^3 - 2t - 15t$ 5. $2x^2 + 7/2x + 3/4$ [CBSE 2020,12]

1. $4x^2 - 3x - 1$

Solution:

$$\begin{aligned}\text{Let } p(x) &= 4x^2 - 3x - 1 \\ &= 4x^2 - 4x + x - 1 \\ &= 4x(x - 1) + 1(x - 1) \\ &= (x - 1)(4x + 1)\end{aligned}$$

The zeroes of $p(x)$ are given by $p(x) = 0$

So, the value of $4x^2 - 3x - 1$ is zero when $x - 1 = 0$ and $4x + 1 = 0$ i.e, when $x = 1$ or $x = -\frac{1}{4}$

So, the zeroes of $p(x)$ are **1 and $-\frac{1}{4}$** .

Verification:

Sum of zeroes = - (coefficient of x) $\div$ coefficient of x^2

$$\begin{aligned}\alpha + \beta &= -b/a \\ \Rightarrow 1 - \frac{1}{4} &= -(-\frac{3}{4}) \\ \Rightarrow (4 - 1)/4 &= \frac{3}{4} \\ \Rightarrow \frac{3}{4} &= \frac{3}{4}\end{aligned}$$

Product of the zeroes = constant term $\div$ coefficient of x^2

$$\begin{aligned}\alpha\beta &= c/a \\ \Rightarrow 1(-\frac{1}{4}) &= -\frac{1}{4} \\ \Rightarrow -\frac{1}{4} &= -\frac{1}{4}\end{aligned}$$

Hence, verified.

2. $3x^2 + 4x - 4$

Solution:

$$\begin{aligned}\text{Let } p(x) &= 3x^2 + 4x - 4 \\ &= 3x^2 + 6x - 2x - 4 \\ &= 3x(x + 2) - 2(x + 2) \\ &= (x + 2)(3x - 2)\end{aligned}$$

The zeroes of $p(x)$ are given by $p(x) = 0$

So, the value of $3x^2 + 4x - 4$ is zero when $x + 2 = 0$ and $3x - 2 = 0$ i.e, when $x = -2$ or $x = \frac{2}{3}$

So, the zeroes of $p(x)$ are **-2 and $\frac{2}{3}$** .

Verification:

Sum of zeroes = - (coefficient of x) ÷ coefficient of x^2

$$\alpha + \beta = -b/a$$

$$\Rightarrow -2 + 2/3 = -4/3$$

$$\Rightarrow (-6 + 2)/3 = -4/3$$

$$\Rightarrow -4/3 = -4/3$$

Product of the zeroes = constant term ÷ coefficient of x^2

$$\alpha\beta = c/a$$

$$\Rightarrow -2(2/3) = -4/3$$

$$\Rightarrow -4/3 = -4/3$$

Hence, verified.

3. $5t^2 + 12t + 7$

Solution:

$$\text{Let } p(t) = 5t^2 + 12t + 7$$

$$= 5t^2 + 7t + 5t + 7$$

$$= t(5t + 7) + 1(5t + 7)$$

$$= (5t + 7)(t + 1)$$

The zeroes of $p(t)$ are given by $p(t) = 0$

So, the value of $5t^2 + 12t + 7$ is zero when $5t + 7 = 0$ and $t + 1 = 0$ i.e, when $t = -7/5$ or $t = -1$

So, the zeroes of $p(t)$ are **$-7/5$ and -1** .

Verification:

Sum of zeroes = - (coefficient of t) ÷ coefficient of t^2

$$\alpha + \beta = -b/a$$

$$\Rightarrow -7/5 - 1 = -12/5$$

$$\Rightarrow (-7 - 5)/5 = -12/5$$

$$\Rightarrow -12/5 = -12/5$$

Product of the zeroes = constant term ÷ coefficient of t^2

$$\alpha\beta = c/a$$

$$\Rightarrow -7/5(-1) = 7/5$$

$$\Rightarrow 7/5 = -7/5$$

Hence, verified.

4. $t^3 - 2t^2 - 15t$

Solution:

$$\text{Let } p(t) = t^3 - 2t^2 - 15t$$

Taking t common

$$= t\{t^2 - 2t - 15\}$$

$$= t\{t^2 - 5t + 3t - 15\}$$

$$= t\{t(t - 5) + 3(t - 5)\}$$

$$= t(t - 5)(t + 3)$$

The zeroes of $p(t)$ are given by $p(t) = 0$

So, the value of $t^3 - 2t^2 - 15t$ is zero when $t = 0$, $t + 3 = 0$ and $t - 5 = 0$

$$\Rightarrow t = 0 \text{ or } t = -3 \text{ or } t = 5$$

Verification:

Sum of zeroes = coefficient of $t^2 \div$ coefficient of t^3

$$\alpha + \beta + \gamma = -b/a$$

$$0 + (-3) + 5 = -(-2/1)$$

$$-3 + 5 = 2$$

$$2 = 2$$

Sum of product of zeroes taken two at a time = coefficient of $t \div$ coefficient of t^3

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$$

$$(0)(-3) + (-3)(5) + (0)(5) = -15/1$$

$$0 - 15 + 0 = -15$$

$$-15 = -15$$

Product of all the zeroes = -(constant term) $\div$ coefficient of t^3

$$\alpha\beta\gamma = -d/a$$

$$(0)(-3)(5) = 0$$

$$0 = 0$$

Hence, verified.

5. $2x^2 + 7/2x + 3/4$

Solution:

$$\text{Let } p(x) = 2x^2 + 7/2x + 3/4$$

$$= 8x^2 + 14x + 3$$

[Multiply by 4]

$$= 8x^2 + 12x + 2x + 3$$

$$= 4x(2x + 3) + 1(2x + 3)$$

$$= (2x + 3)(4x + 1)$$

The zeroes of $p(x)$ are given by $p(x) = 0$

So, the value of $2x^2 + 7/2x + 3/4$

is zero when $2x + 3 = 0$ and $4x + 1 = 0$ i.e, when $x = -3/2$ or $x = -1/4$

So, the zeroes of $p(x)$ are **$-3/2$ and $-1/4$**

Verification:

Sum of zeroes = - (coefficient of x) ÷ coefficient of x^2

$$\alpha + \beta = -b/a$$

$$\Rightarrow -3/2 + (-1/4) = -7/2 \times 2$$

$$\Rightarrow -3/2 - 1/4 = -7/4$$

$$\Rightarrow (-6 - 1)/4 = -7/4$$

$$\Rightarrow -7/4 = -7/4$$

Product of the zeroes = constant term ÷ coefficient of x^2

$$\alpha\beta = c/a$$

$$\Rightarrow -3/2(-1/4) = 3/2 \times 4$$

$$\Rightarrow 3/8 = 3/8$$

Hence, verified.

EX 2.4

1. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also, find the zeroes of these polynomials by factorisation.

$$(i) \quad \frac{-8}{3}, \frac{4}{3}$$

$$(ii) \quad \frac{21}{8}, \frac{5}{16}$$

$$(iii) \quad -2\sqrt{3}, -9$$

$$(iv) \quad \frac{-3}{2\sqrt{5}}, -\frac{1}{2}$$

[CBSE 2016,10,09]

Solution:

(i) Given: sum of zeroes (S) = $-\frac{8}{3}$

and product of zeroes (P) = $\frac{4}{3}$

$\therefore$ Required quadratic polynomial is $p(x) = x^2 - Sx + P$

$$= x^2 + (-\frac{8}{3})x + \frac{4}{3}$$

$$= x^2 - \frac{8x}{3} + \frac{4}{3}$$

i.e., $3x^2 + 8x + 4$ [Multiply by 3]

Using factorisation method,

$$p(x) = 3x^2 + 6x + 2x + 4$$

$$= 3x(x + 2) + 2(x + 2)$$

$$= (x + 2)(3x + 2)$$

The zeroes of $p(x)$ are given by $p(x) = 0$

$$(x + 2)(3x + 2) = 0$$

$$x + 2 = 0 \text{ or } 3x + 2 = 0$$

$$x = -2 \text{ or } x = -\frac{2}{3}$$

Hence, the zeroes of $p(x)$ are -2 and $-\frac{2}{3}$.

Solution:

(ii) Given that, the sum of zeroes (S) = $\frac{21}{8}$

and the Product of zeroes (P) = $\frac{5}{16}$

$\therefore$ Required quadratic polynomial is $p(x) = x^2 - Sx + P$

$$= x^2 - \frac{21x}{8} + \frac{5}{16}$$

i.e., $16x^2 - 42x + 5$ [Multiply by 16]

Using factorisation method,

$$\begin{aligned}
 p(x) &= 16x^2 - 40x - 2x + 5 \\
 &= 8x(2x - 5) - 1(2x - 5) \\
 &= (2x - 5)(8x - 1)
 \end{aligned}$$

The zeroes of $p(x)$ are given by $p(x) = 0$

$$(2x - 5)(8x - 1) = 0$$

$$(2x - 5) = 0 \text{ or } (8x - 1) = 0$$

$$x = 5/2 \text{ or } x = 1/8$$

Hence, the zeroes of $p(x)$ are $5/2$ and $1/8$.

Solution:

(iii) Given: sum of zeroes (S) = $-2\sqrt{3}$ and product of zeroes (P) = -9

Required quadratic polynomial is $p(x) = x^2 - Sx + P$

$$p(x) = x^2 + 2\sqrt{3}x - 9$$

Using factorisation method, $p(x) = x^2 + 3\sqrt{3}x - \sqrt{3}x - 9$

$$= x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3})$$

$$= (x + 3\sqrt{3})(x - \sqrt{3})$$

The zeroes of $p(x)$ are given by $p(x) = 0$

$$(x + 3\sqrt{3})(x - \sqrt{3}) = 0$$

$$(x + 3\sqrt{3}) = 0 \text{ or } (x - \sqrt{3}) = 0$$

$$x = -3\sqrt{3} \text{ or } x = \sqrt{3}$$

Hence, the zeroes of $p(x)$ are $-3\sqrt{3}$ and $\sqrt{3}$

Solution:

(iv) Given that, the sum of zeroes (S) = $-3/2\sqrt{5}$

Product of zeroes (P) = $-1/2$

∴ Required quadratic polynomial is $p(x) = x^2 - Sx + P$

$$= x^2 - (-3/2\sqrt{5})x + (-1/2)$$

$$= x^2 + 3x/2\sqrt{5} - 1/2$$

$$\text{i.e., } 2\sqrt{5}x^2 + 3x - \sqrt{5} \quad [\text{Multiply by } 2\sqrt{5}]$$

Using factorisation method,

$$p(x) = 2\sqrt{5}x^2 + 5x - 2x - \sqrt{5}$$

$$= \sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5})$$

$$= (2x + \sqrt{5})(\sqrt{5}x - 1)$$

The zeroes of $p(x)$ are given by $p(x) = 0$

$$(2x + \sqrt{5})(\sqrt{5}x - 1) = 0$$

$$(2x + \sqrt{5}) = 0 \text{ or } (\sqrt{5}x - 1) = 0$$

$$x = -\sqrt{5}/2 \text{ or } x = 1/\sqrt{5}$$

Hence, the zeroes of $p(x)$ are $-\sqrt{5}/2$ and $1/\sqrt{5}$.